Learning in Asymptotically Behaving Neural Networks *

Nikitas J. Dimopoulos, Don Radvan and W. A. Keddy
Department of Electrical and Computer Engineering
University of Victoria
Victoria, B.C., CANADA

Abstract

The asymptotic behavior of Neural Networks modeled as a set of nonlinear differential equations of the form $T\dot{X} + X = W \cdot \mathbf{f}(X) + \mathbf{b}$ where X is the neural membrane potential vector, W is the network connectivity matrix, and $\mathbf{f}(X)$ is the nonlinearity (an essentially sigmoid function), has been studied in [4]. This behavior depends solely on the topology of the network.as expressed by the connectivity matrix W.

In this work, we present some results of Hebbian Learning by neural networks exhibiting asymptotic behavior as stipulated by their connectivity matrices W.

We also present the simulator that has been developed specifically for this type of neural networks as well as typical examples.

1 Introduction

Neural Networks have been studied for several decades now, with the recent flurry of activity corresponding to the ability of constructing hardware analogs capable of mimicking aspects of the behavior of the central nervous system. Thus Mead [12] has constructed a 48X48 electronic analog of the retina that is capable of calculating time derivatives and hence model cells that look for motion. Psaltis et al [14], have implemented learning networks using holograms. Newcomb et al [11] have produced circuits that capture the temporal and spatial activity of multiinput neural elements, while Moopenn et al [15] have constructed electronic analogs of associative memory.

Several studies have been undertaken in the past years attempting to analyze the behavior of systems comprising a large number of interelated and interacting elementary neurons. Thus Morishita et al[16] have studied networks of mutually inhibiting neurons. Hopfield [9,10] has studied the collective properties of systems consisting of neurons with binary or graded response. Properties such as absolute stability were proven for networks having symmetric interconnection matrices. Cohen and Grossberg [3] have also studied the absolute stability of global pattern formation in competitive neural networks. Again a prerequisite of absolute stability for such systems is the symmetry of the interconnection matrix. In [5,4] the behavior of a generalized class of neural networks was examined. Such networks are composed of distinct classes of neurons and the manner that these classes interconnect provides for stability.

2 Description of the Network

We are interested in neural networks composed of k classes of neurons, each class having $n_i; i = 1, 2, \ldots k$ neurons and adhering to the microscopic and macroscopic connectivity principles as discussed in [6]. These principles essentially state that a neural network is composed of a distinct and limited number of classes of similar neurons, and that neurons from a given class interconnect with neurons of other classes in a predetermined way that is invariant across a species (macroscopic connectivity principle). The strength of these interconnections vary from member to member, and they are subject to learning. Neurons connect to a small percentage of neighboring neurons, subject to the macroscopic connectivity principle (microscopic connectivity principle).

Such neural networks are described by systems of differential equations of the form

$$T\dot{X} + X = W \cdot \mathbf{f}(X) + \mathbf{b}$$

where

$$X = [X_1 X_2 \ldots X_k]^T$$
$$= [x_1 x_2 \ldots x_{n_1}, x_{n_1+1}, \ldots x_{n_1+n_2}, \ldots x_{N-n_{k-1}+1} \ldots x_N]^T$$

is the neural membrane potential vector,

$$N = \sum_{i=1}^{k} n_i$$

is the total number of neurons in the network

*This work was supported by the Natural Science and Engineering Research Council of Canada under Grant #A1337.

$$W = \begin{bmatrix} W_{11} & W_{12} & \dots & W_{1k} \\ \vdots & & & \\ W_{k1} & W_{k2} & \dots & W_{kk} \end{bmatrix}$$

is the network connectivity matrix.

$\mathbf{f}(X)$ belongs to the class of the neuromime or activation functions.

$T = \text{diag}\,[\tau_i] \qquad i = 1, 2, \dots N$

is the membrane time constant, a positive matrix and finally $\mathbf{b}$ is a constant vector.

Of particular importance in formulating the system of differential equations are the connectivity matrix W and the set of neuromime functions.

The connectivity matrix W, reflects the topology of the neural network in question. Each one of the submatrices W_{ij} may be either positive, negative or zero depending on whether neurons from class j excite, inhibit or have no direct effect whatsoever on neurons in class i.

As an example to clarify the point, take the case of networks topologically similar to the cerebellum [7,1,2]. Such networks are comprised of four neural classes $\aleph_1$, $\aleph_2$, $\aleph_3$ and $\aleph_4$ corresponding to Purkinje, basket, Golgi and granule cells. $\aleph_1$, $\aleph_2$ and $\aleph_3$ are classes of inhibitory neurons while $\aleph_4$ is a class of excitatory ones.

The connectivity matrix of such a network can be easily established from the organization of the cerebellum [7] and has the form

$$W = \begin{bmatrix} 0 & W_{12} & 0 & W_{14} \\ 0 & 0 & 0 & W_{24} \\ 0 & 0 & 0 & W_{34} \\ 0 & 0 & W_{43} & 0 \end{bmatrix}$$

with W_{12}, $W_{43} < 0$ and W_{14}, W_{24}, $W_{34} > 0$. This reflects the fact that, for example, the basket cells inhibit the Purkinje cells ($W_{12} < 0$) and that there is no connection between basket and Golgi cells ($W_{23} = W_{32} = 0$).

The neuromime or activation functions, on the other hand, describe the frequency behavior of the spike train on the axon as a function of the hillok potential. Generally speaking, spikes appear on the axon once the hillok potential exceeds a certain threshold [8], and their frequency increases with the potential. The neuromime functions, as defined below incorporate these properties, namely they are positive non-decreasing functions with a threshold.

Definition 1. The class $\aleph$ (the neuromime functions) is defined as follows

$\aleph = \{\mathbf{f} | \mathbf{f} : R^N \rightarrow R^M_+$, $\mathbf{f}$ continuous, $\mathbf{f}$ monotonically non-decreasing, satisfying a Lipschitz condition and $\exists\, \theta \in R^N$ such that $\mathbf{f}(\theta) = 0\}$.

The postulation of Lipschitzian behavior for the neuromime functions, according to Def. 1, guarantees the uniqueness of solutions for the system of differential equations.

As it turns out, neural structures found in the central nervous system of several species can be modelled effectively by networks as described above. One of the goals of our analysis is to establish fundamental properties of such structures that contribute to the asymptotic behavior. We were able to define a general class of such networks which we denote by $\wp$ and which exhibit asymptotic behavior. Neural networks modeling the cerebellum is but one element of this class.

In [4] we have proven the following theorem that establishes this class of Neural Networks exhibiting asymptotic behavior.

Theorem The system

$\dot{X} + TX = W\mathbf{f}(X) + K$

with $\mathbf{f}(x) \in \aleph^*$ and where the connectivity matrix W has all its positive entries located above the main diagonal belongs to class $\wp$.

3 Hebbian Learning

Hebbian Learning was postulated by D. O. Hebb [13] who observed that changes happen to natural networks as they learn, and that these changes depend on the firing patterns of the neurons involved. The Hebb postulate is "When the axon of cell A is near enough to excite a cell B and repeatedly or persistently takes part in firing it, some growth process takes place in one or both cells such that A's efficiency as one of the cells firing B increases". One can write the following general rule of modifying the synaptic weights in response to the activity patterns developing in the network.

$$\frac{\partial w_{ij}}{\partial t} = K \cdot f(x_j) \cdot [-E \cdot w_{ij} + h(x_i)]$$

where K and E are constants, x_i is the state of neuron i, $f(x_j)$ is the valuation function for neuron j (the afferent neuron) and $h(x_i)$ is the suprathreshold value for neuron i (the efferent neuron) and is defined thus

$$h(x_i) = \begin{cases} 0 & \text{if} \quad x_i < t_i \\ x_i - t_i & otherwise \end{cases}$$

and t_i is the threshold for neuron i. Observe that functions $f(x_i)$ and $h(x_i)$ are different. This is explained if one observes that at the synapse, the only information from the afferent neuron is its firing frequency which is given by the valuation function $f(x_i)$, while the membrane potential is the only information available from the efferent neuron, and this is given by the suprathreshold function $h(x_i)$.

The above equation describes accurately the learning behavior in case of excitatory afferent neurons. Indeed, if $f(x_i) = 0$, then

$$\frac{\partial w_{ij}}{\partial t} = 0$$

and there is no change in the weight w_{ij}; presumably any activity in the efferent neuron does not, at this moment, depend on the afferent neuron. On the other hand if $f(x_i) \neq 0$, then depending on whether the efferent neuron is active or not, one obtains two different behaviors for the weight. Indeed, if the efferent neuron is not active, i.e. $h(x_i) = 0$ then

$$\frac{\partial w_{ij}}{\partial t} = -KE \cdot f(x_j) \cdot w_{ij}$$

with a solution that tends to zero with time. This agrees with the fact that if the efferent neuron is not active while the afferent is, then the influence of the afferent to the efferent should diminish. On the other hand if the efferent is firing, i.e. $h(x_i) > 0$, then

$$\frac{\partial w_{ij}}{\partial t} = K \cdot f(x_j) \cdot [-E \cdot w_{ij} + h(x_i)]$$

and the weight tends to increase with time towards $K \cdot f(x_j) \cdot h(x_i)$in accordance with the Hebbian postulate.

The discussion above applies well to the case where the afferent neuron is an excitatory one ($w_{ij} > 0$). In case where the afferent neuron is an inhibitory one, the weight adaptation equation is modified to conform with the learning where the magnitude of the weight decreses if the efferent neuron is firing while it increases (i.e. the synapse becomes stronger) if it is is silent.

$$\frac{\partial w_{ij}}{\partial t} = K \cdot f(x_j) \cdot [-E \cdot w_{ij} + h^*(x_i)]$$

where K and E are constants, x_i is the state of neuron i, $f(x_j)$ is the valuation function for neuron j (the afferent neuron) and $h^*(x_i)$ is the subthreshold value for neuron i (the efferent neuron) and is defined thus

$$h^*(x_i) = \begin{cases} 0 & \text{if} \quad x_i > t_i \\ x_i - t_i & otherwise \end{cases}$$

We have incorporated the above Hebbian Learning in our simulator, discussed in more detail in Section 4 below, and we have trained successfully some simple networks. Specifically, the network that we used comprises two classes of neurons, one is excitatory and the other inhibitory. The excitatory class feeds the inhibitory one while the neurons in the inhibitory class have lateral connections. Two inputs are used. One input feeds the class of the excitatory neurons and carries patterns from the training set. The other feeds the class of the inhibitory neurons and is used as an association input so that the response patterns correspond with the desired output. The topology of the network used is given in Fig. 1. while its connectivity matrix is as follows:

$$W = \begin{bmatrix} 0 & 0 \\ W_{12} & W_{22} \end{bmatrix} \qquad ; W_{12} > 0, \quad W_{22} < 0$$

As it can be seen, such a network conforms with Theorem 1. and therefore exhibits asymptotic behavior. Also because the learning rules, introduces earlier, preserve the polarity of the weights, it is guaranteed that the trained network will exhibit asymptotic behavior.

Figures 2. gives the training sets used. Figures 3, 4, and 5 give the response of the trained network to stimuli from the training class of inputs (class I), while the association class of inputs (class J) is silent. As it can be seen, we have used an XOR problem, where when both input patterns are active the output is silenced, while either single pattern elicits the same response. The training was quite efficient, it took 100 iterations for the weights to stabilize, and a run time of less than 90 seconds on a SUN 3/60 machine.

4 Simulator

As we mentioned earlier, we have constructed a simulator, that provides us with a useful tool for studying the effect of the structure on the behavior of neural networks, and allows us to experiment with learning. The simulator is written in FORTRAN with an X-windows user friendly interface that can be used to define the topology of the network, define the input patterns run and graphically observe the results of both the training and the trained network. Our simulator runs currently on a SUN environment, but we are in the process of porting it to an IBM (3090 / RT) environment. Figure 6 gives a sample of the user interface.

5 Discussion

In this work, we presented some recent results of our attempts to introduce learning for a class of neural networks that has been proven to be asymptotically stable, and can be used to model several existing structures in the Central Nervous System (e.g. cerebellum). Specifically, we discussed the structure of this class of asymptotically behaving neural networks, introduced a hebbian learning rule that can be used to modify both the inhibitory and excitatory synapses, and used this rule to train a simple network from this class in a XOR problem. We also presented the simulator and its user interface that we are developing for the study of such problems. We are further investigating the suitability of different topologies in general pattern recognition problems.

References

[1] J. C. Eccles, M. Ito, J. Szentaghothai *The Cerebellum as a Neuronal Machine* Springer Verlag, (1967)

[2] D. Marr "A theory of Cerebellar Cortex" *J. Physiol.* 203, pp. 437-470 (1969)

[3] M. A. Cohen and S. Grossberg, "Absolute Stability of Global Pattern Formation and Parallel Memory Storage by Competitive Neural Networks", *IEEE Trans. Systems Man and Cybernetics* , Vol. SMC-13, No. 5, pp. 815-826, Sep./Oct. 1983.

[4] N. J. Dimopoulos "A Study of the Asymptotic Behavior of Neural Networks", *IEEE Trans. on Circuits and Systems* , Vol 36, No. 5, pp. 687-694 May 1989.

[5] N. Dimopoulos, R.W. Newcomb, "Stability Properties of a Class of Large Scale Neural Networks", Proceedings of the *1980 IEEE International Symposium on Circuits and Systems* , Houston, pp. 528-530, April 1980.

[6] N. Dimopoulos, "Organization and Stability of a Neural Network Class and the Structure of a Multiprocessor System", Ph.D. Thesis, University of Maryland, 1980.

[7] J.C. Eccles. *The Understanding of the Brain* , McGraw Hill, 1973.

[8] U. an Der Heiden, "Structures of Excitation and Inhibition", in *Theoretical Approaches to Complex Systems* , S. Levine (Ed.), Vol. 21 of the *Lecture Notes in Biomathematics* , Springer-Verlag, pp. 75-88, 1977.

[9] J. J. Hopfield, "Neural networks and physical systems with emergent collective computational abilities", *Proc. Natl. Acad. Sci. USA* , Vol. 79, pp. 2554-2558, Apr. 1982.

[10] J. J. Hopfield, "Neurons with graded response have collective computational properties like those of two-state neurons", *Proc. Natl. Acad. Sci. USA* , Vol. 81, pp. 3088-3092, May 1984.

[11] N. El-Leithy, R.W. Newcomb and M. Zaghloul "A basic MOS neural-type junction. A perspective on neural type microsystems" *Proceedings IEEE First International Conference on Neural Networks* , Vol. III, pp.469-477, Jun. 1987.

[12] Carver Mead, "Silicon Models of Neural Computation" *Proceedings IEEE First International Conference on Neural Networks* , Vol. I, pp.91-106, Jun. 1987.

[13] D.O. Hebb *The Organization of Behavior* John Wiley, New York 1949.

[14] Y. S. Abu-Mostafa and D. Psaltis, "Optical Neural Computers" *Scientific American* , 256(3), 88 (1987).

[15] A. P. Thakoor "Content-Addressable, High Density Memories based on Neural Network Models" *Technical Progress Report* JPL D-4166, Jet Propulsion Laboratory, California Institute of Technology, Pasadena CA., Mar. 1987.

[16] T. Tokura and I. Morishita, "Analysis and Simulation of Double-Layer Neural Networks with Mutually Inhibiting Interconnections", *Biol. Cybernetics* 25, pp. 83-92, 1977.

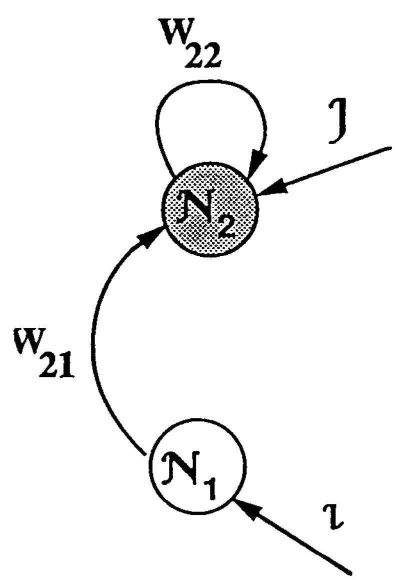


Figure 1. The neural network used for the XOR problem. Class $\aleph_1$ is the input class consisting of excitatory neurons, while class $\aleph_2$ is the output class comprising inhibitory neurons. I and J are input clases. Class I is used for the training patterns, class J is used for the association patterns.

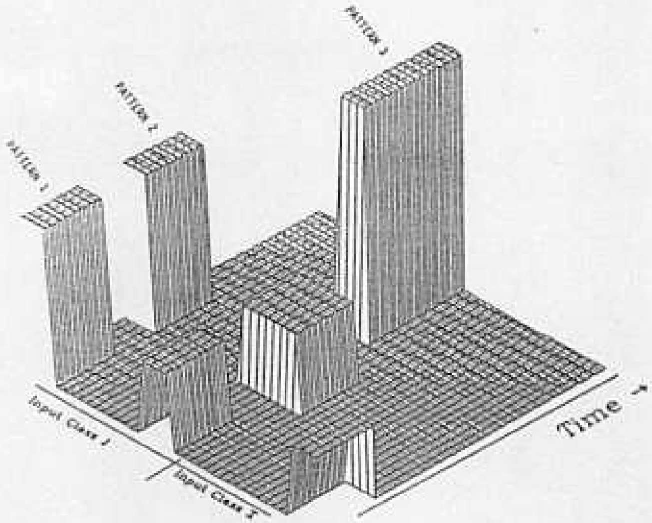


Figure 2. The set of training patterns used.

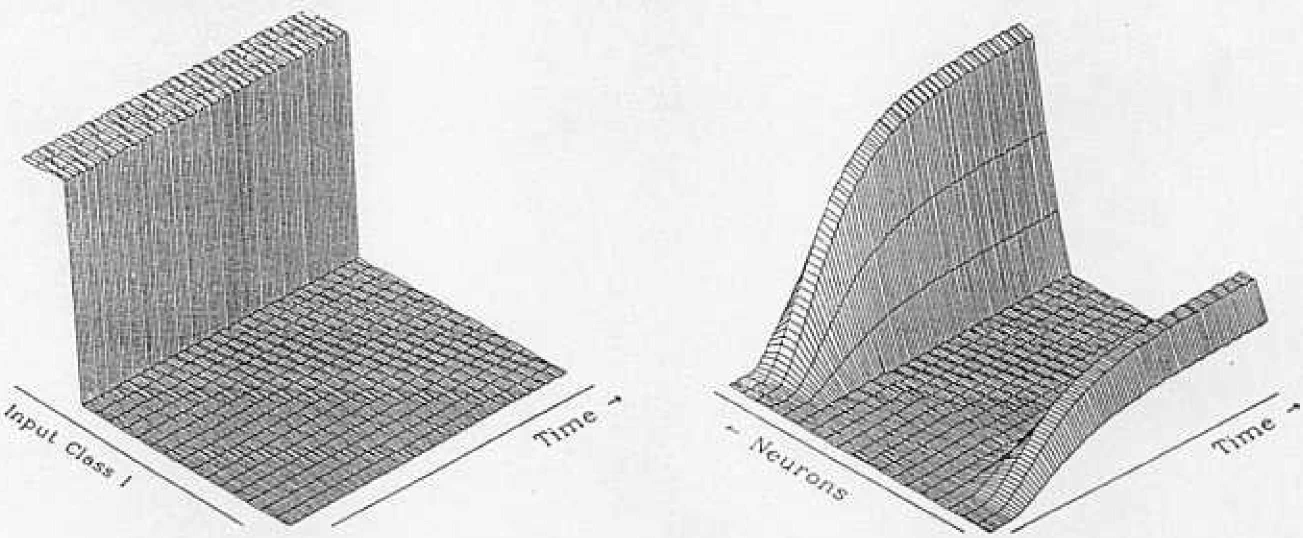


Figure 3. The response of the trained network to a single input on the left hand side of input class I. Input class J is silent.

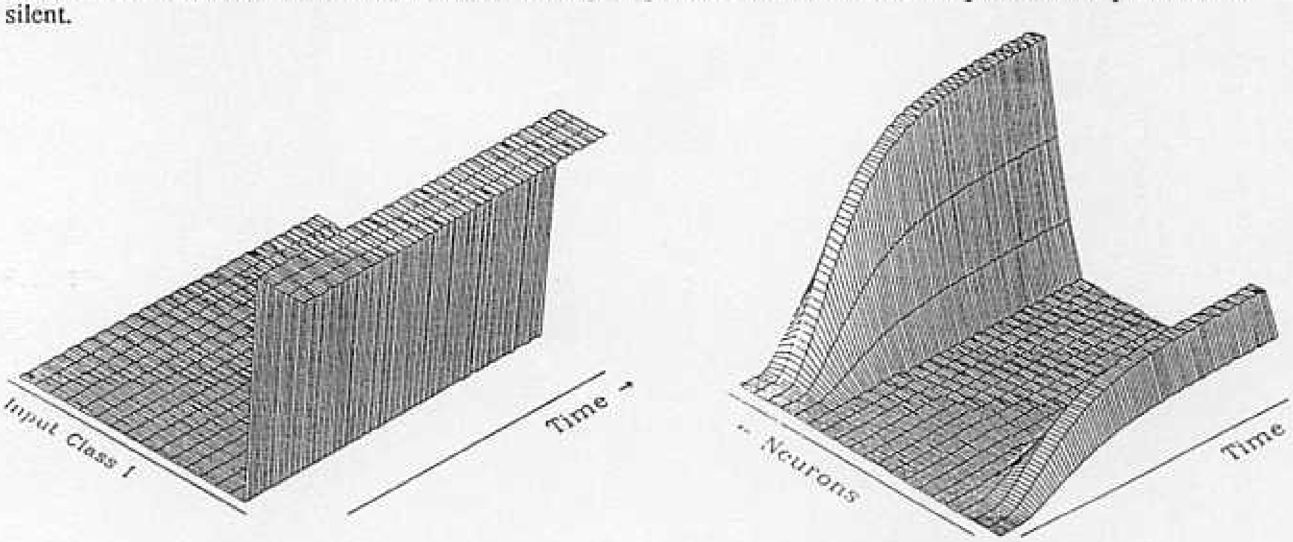


Figure 4. The response of the trained network to a single input on the right hand side of input class I. Input class J is silent.

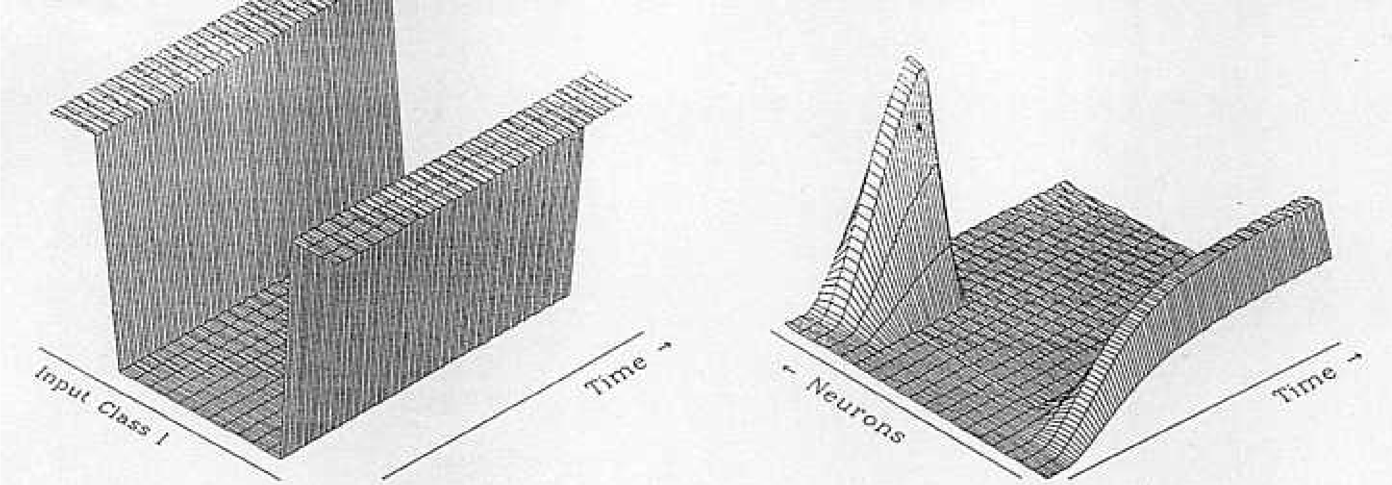
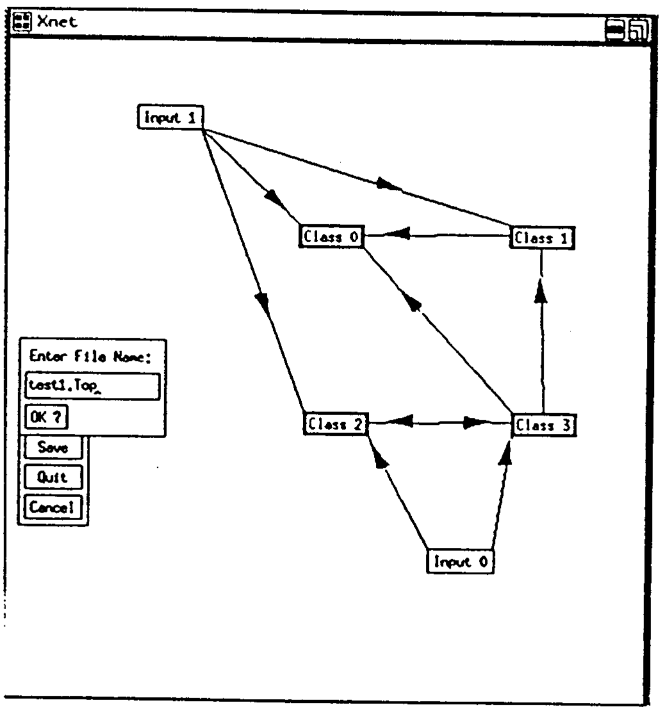
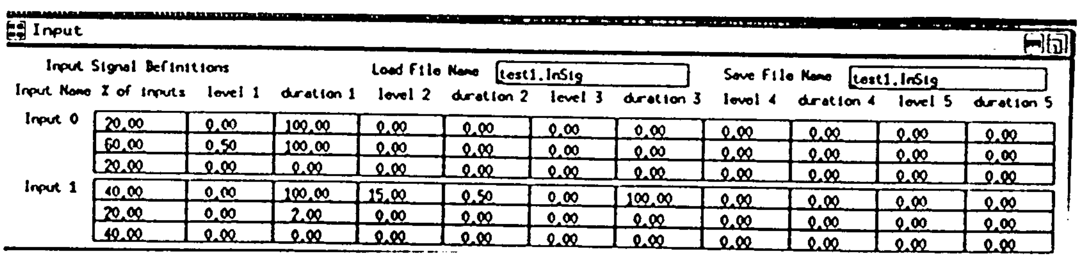


Figure 5. The response of the trained network to inputs on both the left and right hand side of input class I. Input class J is silent. Observe the eventual silencing of the output class.



(a)


Input

Input Signal Definitions   Load File Name test1.InSig   Save File Name test1.InSig

Input Name	% of inputs	level 1	duration 1	level 2	duration 2	level 3	duration 3	level 4	duration 4	level 5	duration 5
Input 0	20.00	0.00	100.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	60.00	0.50	100.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	20.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Input 1	40.00	0.00	100.00	15.00	0.50	0.00	100.00	0.00	0.00	0.00	0.00
	20.00	0.00	2.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	40.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

(b)

Figure 6. (a) XEditNet: editing window depicting topology, and save menu.
(b) XRunNet: Input signal definition window.