HYPERCYCLES. INTERCONNECTION NETWORKS WITH SIMPLE ROUTING STRATEGIES

Nikitas J. Dimopoulos
Electrical and Computer Engineering Department
University of Victoria
Victoria, B.C. Canada

R.D. Rasmussen , G.S. Bolotin , B. F. Lewis, R. M. Manning
Jet Propulsion Laboratory
California Institute of Technology
Pasadena, California

ABSTRACT

In this work, we present the Hypercycles, a class of multidimensional graphs, which are generalizations of the n-cube. These graphs are obtained by allowing each dimension to incorporate more than two elements and a cyclic interconnection strategy. Hypercycles, offer simple routing, and the ability, given a fixed degree, to chose among a number of alternative size graphs. These graphs can be used in the design of interconnection networks for distributed systems tailored specifically to the topology of a particular application.

1.0 Introduction

Message passing concurrent computers such as the Hypercube[11, 16], Cosmic Cube[15], MAX[12, 13], consist of several processing nodes that interact via messages exchanged over communication channels linking these nodes into one functional entity.

There are many ways of interconnecting the computational nodes, the Hypercube, Cosmic Cube, and the Connection Machine[17] having adopted a regular interconnection pattern corresponding to a binary n-dimensional cube, while MAX adopts a less structured, yet unspecified topology.

Several recent studies attempt extensions and generalizations of the basic tenets of the n-cube. Broder et. al. [4] have proposed product graphs[14] of small "basic" graphs. Their prime concern is to synthesize fault tolerant networks with a given degree of coverage. In these multidimensional graphs, they define a single route from a source to a destination, as the product of routes in each of the constituent dimensions. Routing is exhausted in each dimension before another dimension is considered. Bhunyan and Agrawal [3] have introduced the generalized hypercubes (GHC) which are also graph products of fully connected "basic" graphs. The mixed radix system [2] is used to express the properties of these graphs and their routing. Wittie [18] gives a good overview and comparison of several interconnection networks including the spanning bus and dual bus hypercubes. These are essentially binary n-cubes with broadcast busses connecting the processors in each dimension.

The advantages of having a regularly structured interconnection are many-fold, and they have been proven time and again in their being incorporated in many recent designs [6,11,12,13,15,16,17]. In these structures, easy deadlock-free routing [7] can be accomplished by locally computing each successive intermediate node -for a path that originates at a source node and terminates at a destination node- as a function of the current position and the desired destination. Many regular problems (such as the ones found in image processing, physics etc.) have been mapped on such regular structures, and run on the corresponding machines exhibiting significant speedups. In contrast, embedded real-time applications, such as the ones addressed by the MAX project [12, 13], tend to exhibit variable structures that do not necessarily map optimally to an n-cube. In addition, since the size of a binary n-cube is given as 2^n (n being the degree of the graph), it means that a particular configuration cannot be expanded but in predefined quantum steps. For example, if a given embedded application requires a system comprised of 9 nodes, the next larger n-cube with 16 nodes must be chosen. This constitutes a significant increase in resource allocation, especially in the light of the power-mass limited environment of a spacecraft.

Hypercycles[9] can be considered as products of "basic" graphs that allow, as compared to the Generalized Hypercubes (GHC) [3], a richer set of component "basic" graphs ranging in complexity from the simple rings to the fully connected ones used in the GHC. Also, contrary to Broder et. al.[4], we define the component graphs and provide analytical expressions for routing, our aim being twofold:

(a) To provide computer interconnection networks that match the node requirements of a given embedded system. Since our primary target is spacecraft applications which are weight and power limited, (the nodes in a spacecraft computer network are the primary weight contributor rather than the communication media) the exact matching of the node requirements is of paramount importance.

(b) To increase throughput of a given network by providing routing expressions that can be computed analytically (and hence are candidates for VLSI implementation) and which provide a maximum number of alternate paths from a source to a destination. The existence of alternate paths guarantees that a message will not be blocked waiting for its single route to be freed, but it would in turn search for the availability of alternate paths. This strategy also provides for fault protection, since a faulty path can be marked permanently busy, and thus messages can be routed around it. (Such an approach of adaptive routing is applied in the hypercube through the Hyperswitch [6])

The Hypercycles, being regular graphs, retain the advantages of easy routing and regularity. Yet, since we are dealing with a class, rather than isolated graphs, we have the flexibility of adopting any particular graph (from the class) that closely matches the requirements of a given application. Since the graphs belong to the same class, routing is accomplished via the same methods and thus the same hardware can conceivably be used to configure structures with different sizes and topologies.

This work describes such a class of generalized interconnection networks, with routing strategies that are similar to that of the n-cube [4]. Yet, these networks offer richer topologies, and contain both the n-cube and the ring as special cases. While the n-cube is based on representation of nodes in base 2, we generalize by using the mixed radix system representation. Such a representation, includes the binary (and hence the n-cube) as well as the arbitrary base b representation as special cases.

This work is divided into three parts. Section 2.0 introduces the Mixed Radix System, Section 3.0 presents some basic graph terminology and notation, while Section 4.0 introduces the Hypercycles and discusses their properties.

2.0 Mixed Radix Number System

The mixed radix representation [2], is a positional number representation, and it is a generalization of the the standard b-base representation, in that it allows each position to follow its own base independently of the other.

Thus, given a decimal number M factored into r factors $m_1, m_2, m_3 \dots, m_r$ as $M = m_1 \times m_2 \times m_3 \times \dots \times m_r$ then any number $0 \leq X \leq M-1$ can be represented as the following r-tuple

$$(X)_{m_1 m_2 \dots m_r} = x_1 x_2 \dots x_r \,|_{m_1 m_2 \dots m_r}$$

where $0 \leq x_i \leq (m_i - 1)$; $i = 1,2,\dots,r$ and the x_i's are chosen in such a way so as $X = \sum_{i=1}^{r} x_i w_i$ where $w_i = \dfrac{M}{m_1 m_2 \cdots m_i}$

We use the notation $(X)_{m_1 m_2 \dots m_r} = x_1 x_2 \dots x_r \,|_{m_1 m_2 \dots m_r}$ to indicate the radices involved. Since for most cases we shall be dealing with a single set of radices, $m_1, m_2, \dots, m_r$, we shall omit, when obvious, the radix indication from the notation $x_1 x_2 \dots x_r \,|_{m_1 m_2 \dots m_r}$.

As an example, if we chose $M = 10 = 2 \times 5$, then any number between 0 and 9 can be represented with two digits, the first one ranging from 0 to 1, and the second one from 0 to 4. Thus $(6)_{2,5} = 11|_{2,5}$, since $m_1 = 2$, $m_2 = 5$, $w_1 = M / m_1 = 10/2 = 5$, and $w_2 = M / (m_1 m_2) = 1$. Therefore, $(6)_{2,5} = 11|_{2,5} = 1 \times w_1 + 1 \times w_2 = 1 \times 5 + 1 \times 1 = 6|_{10}$.

Similarly, we can see that the mixed radix system representation, is a generalization of the standard base-b system. Indeed, if we select $m_1 = m_2 = \dots = m_r = b$, then $M = b^r$, the corresponding weights w_i become $w_i = M / b^i = b^{r-i}$, therefore, the representation of a number X in base $b \times b \times \dots \times b$ becomes $(X)_{bb \dots b} = \sum_{i=1}^{r} x_i w_i = \sum_{i=1}^{r} x_i b^{r-i}$ which is exactly the representation in base-b.

Based on the above, we proceed now with the presentation of the Hypercycles.

3.0 Graph Notation

An undirected graph G is defined as the following tuple: $\mathfrak{a} = (\mathrm{N}, \mathcal{E})$, where N is the set of nodes (vertices) $\mathrm{N} = \{ \alpha_i \; ; i = 1,2,...N \}$, and $\mathcal{E}$ the set of edges defined as

$$\mathcal{E} = \left\{ e_{i j_i} = \left(\alpha_i \, , \beta_{j_i} \right) j_i = 1,2,...,d_i \; ; \; i = 1,2,...,N \right\}$$

with $\alpha_i \, , \beta_{j_i} \in \mathrm{N}$ and where d_i is the degree of node α_i (i.e. the number of edges incident at a particular node). The degree of a graph, denoted $d(\mathfrak{a})$, is defined as the maximum of the node degrees. A walk in $\mathfrak{a}$ [5] is a sequence of edges $e_1 e_2 \ldots e_l$, such that if $e_i = (\alpha_i, \alpha_{i+1})$ then $e_{i+1} = (\alpha_{i+1}, \alpha_{i+2})$ and $e_i \in \mathcal{E}$. The length l of such a walk is defined as the number of edges occurring in it. The distance, $\mathbf{dis}(\gamma,\delta)$, between nodes γ and δ is defined as the shortest walk between γ and δ if any, otherwise, $\mathrm{dis}(\gamma,\delta) = \infty$. The diameter of a graph, denoted by k, is defined as the maximum distance between any pair of nodes. A graph is regular, if all nodes have the same degree.

4.0 Hypercycles.

We define now an r- dimensional Hypercycle, as the following regular undirected graph:

$$\mathfrak{a}^{\rho}_{m} = \left\{ \mathrm{N}^{\rho}_{m} \, , \mathcal{E}^{\rho}_{m} \right\}$$

where $m = m_1, m_2, m_3, \ldots, m_r$ a mixed radix, $\rho = \rho_1, \rho_2, \ldots, \rho_r \; ; \rho_i \leq m_i / 2$ the connectivity vector, determining the connectivity in each dimension which ranges from a cycle ($\rho_i = 1$) to the fully connected ($\rho_i = \lfloor m_i /2 \rfloor$), and $\mathrm{N}^{\rho}_{m} = \{0,1,2,...,M-1\}$. Each node $\alpha \in \mathrm{N}^{\rho}_{m}$ is represented in the mixed radix system $m_1, m_2, m_3, \ldots, m_r$ as $(\alpha)_{m_1 m_2 \ldots m_r} = \alpha_1 \alpha_2 \ldots \alpha_r$. The set of edges $(\mathrm{N}^{\rho}_{m})^2 \supseteq \mathcal{E}^{\rho}_{m}$ is defined in the following way. Given $\alpha \, , \beta \in \mathrm{N}^{\rho}_{m}$ and if $\alpha \, , \beta$ are represented as $(\alpha)_{m_1 m_2 \ldots m_r} = \alpha_1 \alpha_2 \ldots \alpha_r$ and $(\beta)_{m_1 m_2 \ldots m_r} = \beta_1 \beta_2 \ldots \beta_r$. Then $(\alpha \, , \beta) \in \mathcal{E}^{\rho}_{m}$ if and only if there exists $1 \leq j \leq r$ such that

$$\beta_j = (\alpha_j \pm \xi_j)\,\mathrm{mod}\, m_j \quad \text{with } 1 \leq \xi_j \leq \rho_j$$
$$\text{and } \alpha_i = \beta_i \; ; i \neq j$$

Observe that since $\beta_j = (\alpha_j \pm \beta_j)\,\mathbf{mod}\, m_j$, $\mathcal{E}^{\rho}_{m}$ is symmetric.

Example: Suppose that $M = 12 = 4 \times 3 = m_1 \times m_2$. and $\rho = 1,1 = \rho_1, \rho_2$ .Therefore, for the graph generated by using the above mixed radix and connectivity vector, node $(1)_{4,3} = 01 = \alpha_1 \alpha_2$, is distance one away from node $(10)_{4,3} = 31 = \beta_1 \beta_2$. Indeed, according to our definition, $\beta_1 = 3 = = (0 - 1) \bmod 4 = (\alpha_1 - \xi_1) \bmod m_1$, and $\alpha_2 = \beta_2 = 1$.

The previously defined Hypercycles, are regular graphs of degree[§]

$$d = \sum_{i=1}^{r} f(m_i, \rho_i) \quad \text{where}$$

$$f(m_i, \rho_i) = \begin{cases} 2\rho_i & \text{if} \quad 2\rho_i < m_i \\ m_i - 1 & \text{if} \quad 2\rho_i = m_i \end{cases}$$

and diameter k [¶] [†]. $k = \displaystyle\sum_{i=1}^{r} \left\lceil \frac{\lfloor m_i /2 \rfloor}{\rho_i} \right\rceil$

It is easy to see that the n-cube is a Hypercycle, obtained with $M = 2 \times 2 \times \cdots \times 2 = 2^n$ and $\rho = 1,1,1,...,1$. Both the diameter and the degree of the n-cube are equal to n .

4.1 Routing

Hypercycles, have routing properties that are similar to those of the n-cube. From the definition of the Hypercycle, nodes
$(\alpha)_{m_1 m_2 \ldots m_i \ldots m_r} = \alpha_1 \alpha_2 \ldots \xi \ldots \alpha_r$ and
$(\alpha^*)_{m_1 m_2 \ldots m_i \ldots m_r} = \alpha_1 \alpha_2 \ldots \xi \ldots \alpha_r$

are at most distance $\left\lceil \frac{\lfloor m_i /2 \rfloor}{\rho_i} \right\rceil$ away. A walk, from node α to node α^*, of length $l_{max} = \left\lceil \frac{\lfloor m_i /2 \rfloor}{\rho_i} \right\rceil$ can be constructed as follows:

$$\alpha_1 \alpha_2 \ldots \alpha_i \ldots \alpha_r \, , \alpha_1 \alpha_2 \ldots \xi_1 \ldots \alpha_r \, , \alpha_1 \alpha_2 \ldots \xi_2 \ldots \alpha_r \, , \ldots, \alpha_1 \alpha_2 \ldots \xi \ldots \alpha_r \, .$$

such that[¶]

$$\xi_{j_i +1} = \begin{cases} \xi_{j_i} + \rho_i & \text{if} \quad \left[(\xi - \xi_{j_i})\,\mathbf{mod}\, m_i = \left| \xi_{j_i}, \xi \right| \right] > \rho_i \\ \xi_{j_i} - \rho_i & \text{if} \quad \left[(\xi_{j_i} - \xi)\,\mathbf{mod}\, m_i = \left| \xi_{j_i}, \xi \right| \right] > \rho_i \\ \xi & \text{if} \quad \left| \xi_{j_i}, \xi \right| \leq \rho_i \end{cases}$$

$$\xi_0 = \alpha_i \qquad\qquad \xi_{l_{max}} = \xi$$

We call the length l_{max} of such a walk, the distance along dimension i .
Given an origin $(\alpha)_{m_1 m_2 \ldots m_r} = \alpha_1 \alpha_2 \ldots \alpha_r$
and a destination $(\beta)_{m_1 m_2 \ldots m_r} = \beta_1 \beta_2 \ldots \beta_r$
and if q_i denotes their distance along dimension i , their total distance, denoted as $\mathbf{dis}(\alpha,\beta)$ and defined as the sum of the individual distances along all the dimensions, is given as

$$\mathbf{dis}(\alpha, \beta) = q = \sum_{i=1}^{r} q_i$$

For these nodes, there are a total of [‡]

$$I = \binom{q}{q_1, q_2, \ldots, q_r} = \frac{q!}{q_1! \, q_2! \cdots q_r!}$$

distinct walks of length q that connect them . These paths can be constructed by sequentially modifying the source address, each time substituting a source digit by an intermediate walk digit , as specified in equation (2) above, until the destination is reached. As an example, the following walk connects the source to the **destination.**

$$\textbf{source} = \alpha_1 \alpha_2 \alpha_3 \ldots \alpha_r \, , \; \alpha_1 \xi_1 \alpha_3 \ldots \alpha_r \, , \; \alpha_1 \xi_1 \psi_1 \ldots \alpha_r$$
$$\alpha_1 \xi_2 \psi_1 \ldots \alpha_r \, , \alpha_1 \xi_2 \psi_2 \ldots \alpha_r \, , \ldots, \alpha_1 \xi_2 \beta_3 \ldots \alpha_r \, , \ldots,$$
$$\beta_1 \beta_2 \beta_3 \ldots \beta_r = \textbf{destination}$$

Figure 1., gives an example of two distinct walks of equal length that connect a source to a destination, for a Hypercycle.

---

§ Observe that $|\xi| , |\psi| < m/2$ and $|\xi| \neq |\psi|$ imply that $(\alpha + \xi)\,\mathrm{mod}\, m \neq (\alpha + \psi)\,\mathrm{mod}\, m$. Thus, for every $0 < \xi \leq \rho_i$ $\alpha_1 \alpha_2 \ldots (\alpha_i \pm \xi)\,\mathrm{mod}\, m_i \ldots \alpha_r$ defines two distinct nodes in the graph, for a total of $2\rho_i$ nodes. The only exception occurs when m_i is even. Then, for $\xi = m_i/2$, we have $(\alpha_i + m_i/2)\,\mathrm{mod}\, m_i = (\alpha_i + m_i/2 + m_i/2 - m_i/2)\,\mathrm{mod}\, m_i = (\alpha_i - m_i/2)\,\mathrm{mod}\, m_i$. That is, the same node is defined, and therefore, the total number of nodes reachable, becomes $m_i - 1$.

¶ The function $\lfloor x \rfloor$ denotes the largest integer smaller than or equal to x, while $\lceil y \rceil$ denotes the smallest integer larger than or equal to y.

† This number is calculated as follows: Given that there are r digits in a node address, there are the maximum r differences between a source an a destination address. Also, for each digit (since it is expressed in base m_i) the maximum distance between any two points is given as $d_{\max} = \lfloor m_i / 2 \rfloor$. Thus, because each point is connected to points that are distance ξ ; $1 \leq \xi \leq \rho_i$ away, all the points can be reached in a maximum of $\lceil d_{\max} / \rho_i \rceil$ steps.

¶ Denote as $|\mu, \nu| = \min\{ (\mu - \nu)\,\mathrm{mod}\, m \, , (\nu - \mu)\,\mathrm{mod}\, m \}$, the distance between two integers modulo m .

‡ For the definition of a multinomial number, see [1] pp 32.

4.2 Properties

In this section, we shall present a symmetry property for the Hypercycle .

Theorem. If $m = m_1, m_2, \ldots, m_r$ and $m^* = m_1^*, m_2^*, \ldots, m_r^*$ are two mixed radix bases, $\rho = \rho_1, \rho_2, \ldots, \rho_r$ and $\rho^* = \rho_1^*, \rho_2^*, \ldots, \rho_r^*$, $m^* = \pi(m)$ and $\rho^* = \pi(\rho)$ (where π is a permutation), then the graphs G_m^{ρ} and $\mathrm{G}_{m^*}^{\rho^*}$ are isomorphic to each other.

Proof Since m^* is a permutation of m , then

$$\sum_{i=1}^{r} f(m_i) = \sum_{i=1}^{r} f(m_{\pi(i)}) = \sum_{i=1}^{r} f(m_i^*) = \mathrm{d}(\mathrm{G}) \text{ and}$$

$$\sum_{i=1}^{r} \left\lceil \frac{\lfloor m_i/2 \rfloor}{\rho_i} \right\rceil = \sum_{i=1}^{r} \left\lceil \frac{\lfloor m_{\pi(i)}/2 \rfloor}{\rho_{\pi(i)}} \right\rceil = \sum_{i=1}^{r} \left\lceil \frac{\lfloor m_i^*/2 \rfloor}{\rho_i^*} \right\rceil = k$$

Thus, the number of elements, the degrees and the diameters of the graphs G_m^{ρ} and $\mathrm{G}_{m^*}^{\rho^*}$ are identical.

Define now the map g between the nodes of the two graphs in the following manner:

$$N_m^{\rho} \ni \alpha_1 \alpha_2 \ldots \alpha_r \xrightarrow{g} \alpha_1^* \alpha_2^* \ldots \alpha_r^* \in N_{m^*}^{\rho^*}$$

such that $\alpha_j^* = \alpha_{\pi^{-1}(j)}$

The map g is an isomorphism. Indeed, from its construction, it is 1-1 and it can be easily proven that if $\mathrm{dis}(\alpha, \beta) = 1$ then also $\mathrm{dis}(g(\alpha), g(\beta)) = 1$. Indeed, since $\mathrm{dis}(\alpha,\beta) = 1$, there exists an i such that $|\alpha_i - \beta_i| \le \rho_i$ and $\alpha_j = \beta_j$; $j \neq i$. and from the construction of the map g , we have

$$\left|\alpha^*_{\pi(i)} - \beta^*_{\pi(i)}\right| \le \rho_i = \rho^*_{\pi(i)} \text{ and } \alpha^*_{\pi(j)} = \beta^*_{\pi(j)} ; j \neq i$$

and therefore $\mathrm{dis}(g(\alpha), g(\beta)) = 1$.

4.3 Average Distance Calculation

Given an r-dimensional generalized graph G_m^{ρ} , and restricting ourselves in any single dimension i, we can calculate the number of nodes that are distance l away from a source node $(\alpha)_{m_1 m_2 \ldots m_r} = \alpha_1 \alpha_2 \ldots \alpha_r$ as equal to the number of symbols ξ_i that satisfy the relation $(l-1)\rho_i < |\alpha_i, \xi_i| \le l\rho_i$. This number is given as

$$n_l^i = \begin{cases} 2\rho_i & \text{if} \quad l\rho_i < \lceil m_i/2 \rceil \\ (m_i - 1) - 2(l-1)\rho_i & \text{if} \quad \lceil m_i/2 \rceil \le l\rho_i \le \dfrac{m_i - 1 - \rho_i}{2} \\ 0 & \text{otherwise} \end{cases}$$

Given a node $(\alpha)_{m_1 m_2 \ldots m_r} = \alpha_1 \alpha_2 \ldots \alpha_r$ there exist

$$n_1 = \sum_{i=1}^{r} n_1^i \quad \text{nodes at distance one, and in general}$$

$$n_l = \sum_{\substack{l_1, l_2, \ldots, l_r \ge 0 \\ l_1 + l_2 + \ldots + l_r = l}} \prod_{\substack{i=1 \\ l_i \neq 0}}^{r} n_{l_i}^i$$

nodes at distance l .This equation is simplified for the case where $\rho_i = \lfloor m_i/2 \rfloor$ to the following:

$$n_l = \sum_{i_1=1}^{r-l+1} \sum_{i_2=i_1+1}^{r-l+2} \cdots \sum_{i_l=i_{l-1}+1}^{r} (m_{i_1}-1)(m_{i_2}-1)\cdots(m_{i_l}-1)$$

Therefore, the average distance between any two nodes in a Hypercycle G_m^{ρ} can be calculated as

$$\overline{d} = \frac{\sum_{l=1}^{r} l n_l}{m_1 m_2 \cdots m_r}$$

Some typical distances are given in TABLE 1. and TABLE 2.

5.0 Conclusions and Discussion

In this work, we presented the Hypercycle, a class of multidimensional graphs, which are essentially generalizations of the n-cube. These graphs are obtained by allowing each dimension to incorporate more than two elements and a cyclic interconnection strategy.

Although these graphs are not the densest possible, they are attractive, because of their simple routing. Similarly to the n-cube, the destination address is used to sequentially route a message through intermediate nodes as outlined in section 4.1. Also, since the node addresses are represented in a mixed radix as a sequence of r-digits, each one of these digits is processed independently and in parallel with the remaining digits. Thus the hardware involved in the routing can be made fast (because of the parallelism) and simple (since each module need only handle arithmetic $\mathrm{mod} m_i$, as compared to arithmetic $\mathrm{mod} m_1 m_2 \ldots m_r$ needed when all the address digits are necessary as is the case with such networks as the chordal rings [4], or the cube connected cycles [3]).

For these graphs, we have also calculated the average distance between any two nodes. Assuming that the graph is the representation of a multiprocessor, the average distance provides a metric of the communication complexity involved in such a structure. Compare for example the average distances of the 7-cube and the (m = 5 5 5, ρ = 1 1 1) graph. The average distances, as they can be determined from TABLE 2, are 3.528 and 3.629 respectively, both have similar number of nodes, while their diameters are equal to six and seven respectively. TABLES 1. and 2. give a collection of graphs together with their degrees, diameters, number of nodes and average distances.

The graphs presented in this study, are generalizations of some well known graphs such as the binary n-cube, 2- and 3-dimensional meshes, and rings, which are included as special cases. Examples of some special cases are depicted in Figure 1.

The Hypercycle, can be further extended by employing a different connectivity strategy in each dimension. Thus, in its most general form, eqn. 1. can be modified into

$$\beta_j = (\alpha_j + \xi_j) \mathrm{mod} m_j \; ; \; \xi_j \in \left\{ \xi_j^1, \xi_j^2, \ldots, \xi_j^k \right\}$$

provided that the subgraph (projected in dimension j) is connected. Examples of such extensions include the chordal rings [4], cube connected cycles [3] etc. The disadvantage of course, is the increased complexity involved in the routing calculations for that particular dimension.

In conclusion, Hypercycles are generalizations of the basic n-cube and they offer simple routing, and the ability, given a fixed degree, to chose among a number of alternative size graphs. These properties are important for the design of distributed systems of varying size and connectivity, and tailored specifically to the topology of a particular application. These graphs can therefore be used in the design of the interconnection networks of such machines as the MAX [7] or the Hypercube [6,8].

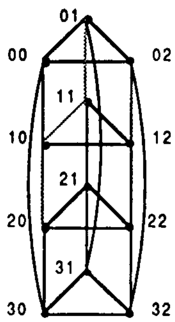
ACKNOWLEDGEMENT

The research described in this paper was carried out by the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration.

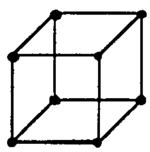
REFERENCES

1. Berge C., *Principles of Combinatorics* Academic Press, New York and London, 1971.
2. Bhuyan, L. N., and D. P. Agrawal, "Design and Performance of Generalized Interconnection Networks" *IEEE Trans. Comput.* Vol. C-32, No. 12, pp. 1081-1090, Dec. 1983.
3. Bhuyan, L. N., and D. P. Agrawal, "Generalized Hypercube and Hyperbus Structures for a Computer Network" *IEEE Trans. Computrs.* Vol. C-33, No. 4. pp. 323-333, (Apr. 1984).
4. A. Broder, D. Dolev, M. Fischer, B. Simons "Efficient Fault Tolerant Routings in Networks" *Proceedings of the 16th Annual ACM Symposium on the Theory of Computing* pp. 536-541, (May 1984).
5. Carlsson, G. E. , J. E. Cruthirds, H. B. Sexton, and C. G. Wright "Interconnection Networks Based on a Generalization of Cube-Connected Cycles" *IEEE Trans. Comput.*, Vol. C-34, No. 8, pp. 769-772, Aug. 1985.
6. E. Chow, H. Madan, J. Peterson "A Real-Time Adaptive Mesage Routing Network for the Hypercube Computer" *Proceedings of the Real-Time Systems Symposium*, pp. 88-96, San Jose CA., (Dec. 1987)
7. Dally, W.J., and C. L. Seitz "Deadlock-Free Message Routing in Multiprocessor Interconnection Networks" *IEEE Trans. Comput.* Vol. C-36, No. 5, pp. 547-553, May 1987.
8. C. Duret " A Nodal Structure Switching Network" *Proceedings Colloque*

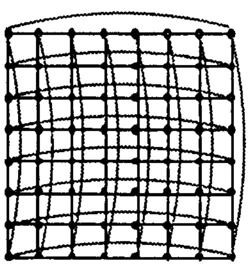
International de Communication Vol. 2, pp. 834-839, Paris, France (May 1979).
9. Feng, T. Y. ,"A Survey of interconnection Networks" *IEEE Computer,* Vol. 14, pp. 12-27, Dec. 1981
10. Imase, M., T. Soneoka, and K. Okada, "Connectivity of Regular Directed Graphs with Small Diameters" *IEEE Trans. Comput.,* Vol. C-34, No. 3, pp. 267-273, Mar. 1985.
11. Peterson, J.C., J. O. Tuazon, D. Lieberman, M. Pniel "The MARK III Hypercube -Ensemble Concurrent Computer" *Proceedings of the 1985 International Conference on Parallel Processing* pp. 71-73, Aug. 20-23 1985.
12. Rasmussen, R. D., G. S. Bolotin, N. J. Dimopoulos, B. F. Lewis, and R. M. Manning "Advanced General Purpose Multicomputer for Space Applications" *Proceedings of the 1987 International Conference on Parallel Processing.* pp. 54-57. (Aug. 1987)
13. Rasmussen, R. D., N. J. Dimopoulos, G. S. Bolotin, B. F. Lewis, and R. M. Manning "MAX: Advanced General Purpose Real-Time Multicomputer for Space Applications" *Proceedings of the IEEE Real Time Systems Symposium* pp. 70-78, San Jose, CA.,(Dec. 1987).
14. G. Sabidussi "Graph Multiplication" *Math. Zeitschr.* Vol. 72, pp. 446-457 (1960).
15. Seitz, C. L. "The cosmic cube" *CACM* vol. 28, pp.22-33, Jan. 1985
16. Tuazon, J. O., J. C. Peterson, M. Pniel, and D. Lieberman "Caltech/JPL MARK II Hypercube Concurrent Processor"*Proceedings of the 1985 International Conference on Parallel Processing* pp. 666-673, Aug. 20-23 1985.
17. Waltz, D. L. "Applications of the Connection Machine" *IEEE Computer* January 1987, pp.85-97
18. L. D. Wittie "Communication Structures for Large Networks of Microcomputers" *IEEE Trans. on Computers* Vol. C-30, No. 4, pp.264-273. (Apr. 1981).



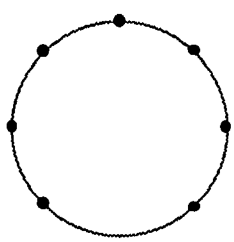
a. Hypercycle α^{11}_{43}



b. Binary 3-cube α^{111}_{222}



c. Illiac IV α^{11}_{88}



d. Ring α^{1}_{7}

Figure 1. Examples of Hypercycles.

TABLE 1.

GRAPH	DEGREE	DIAMETER	NODES	AVERAGE DISTANCE PER NODE
m = 7 ρ = 3	6	1	7	1.0
m = 4 4 ρ = 2 2	6	2	16	1.6
m = 3 3 3 ρ = 1 1 1	6	3	27	2.077
m = 2 2 2 2 2 2 ρ = 1 1 1 1 1 1	6	6	64	3.047

TABLE 2.

GRAPH	DEGREE	DIAMETER	NODES	AVERAGE DISTANCE PER NODE
m = 6 2 ρ = 3 1	6	2	12	1.454
m = 5 3 ρ = 2 1	6	2	15	1.571
m = 5 2 2 ρ = 2 1 1	6	3	20	1.894
m = 4 3 2 ρ = 2 1 1	6	3	24	2.0
m = 4 2 2 2 ρ = 2 1 1 1	6	4	32	2.323
m = 3 3 2 2 r = 1 1 1 1	6	4	36	2.4
m = 3 2 2 2 2 ρ = 1 1 1 1 1	6	5	48	2.723
m = 2 2 17 ρ = 1 1 2	6	6	68	3.403
m = 2 2 3 7 ρ = 1 1 1 1	6	6	84	3.434
m = 3 5 7 ρ = 1 1 1	6	6	105	3.615
m = 5 5 5 ρ = 1 1 1	6	6	125	3.629
m = 2 2 2 2 2 2 2 ρ = 1 1 1 1 1 1 1	7	7	128	3.528
m = 7 7 9 ρ = 1 1 1	6	10	441	5.664