

# Throughput Analysis of A Collision Free Protocol for Local Area Network<sup>1</sup>

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## ABSTRACT

CSMA/CD is a widely accepted access method for local area networks. However, due to the growing number of collisions at heavy load, the throughput of such systems deteriorates. One variation of the CSMA/CD permits waiting stations to contend for the network during the current transmission. At the end of the transmission, the candidate of the next transmission is expected to be chosen thus eliminating the idle period. The network throughput increases correspondingly. This paper describes an analytical method to calculate the throughput of this collision free protocol. The results are then, compared with the throughput of the original CSMA/CD access method.

## I. INTRODUCTION

Local area network is a computer communication network limited to a geographical area, typically within 1 km. Nowadays, it serves as an interoffice communication backbone or as a means to perform some forms of distributed computing. Common Sense Multiple Access (CSMA/CD) [9] is widely used as an access method in local area networks. One such networks is the Ethernet [5].

The CSMA/CD protocol requires each station having messages sense a common channel before and while transmitting to detect possible conflicts. When a conflict occurs, the transmission is aborted. If the channel is busy, the station can persist on sensing (1-persistent) or retry at a later time (nonpersistent). Due to a nonzero propagation delay on the channel, collisions in CSMA/CD exist and reduce the system throughput at heavy loading.

To improve the system throughput, a new collision free variation (CSMA/CF) of the CSMA/CD protocol has been proposed by Wong [11]. This variation employs two separate channels for network acquisition and data transmission. Since conflict resolution for the next transmission can be performed in parallel with the current transmission, the idle period is greatly reduced and consequently the system throughput is increased.

The contention scheme of this collision free protocol will be presented promptly. It is worth to mention that a similar CSMA/CD variation has been proposed with a token ring contention scheme [2] in which token passing delay is incurred.

The CSMA/CF access scheme is described in the following steps, further details can be found in [11].

- Step 1 Sense the contention channel, if it is idle, wait for a fixed interval  $W_1$ , transmit the carrier over the contention channel, otherwise persist on sensing.
- Step 2 Wait for an interval  $W_2$  and check if the carrier is the only one on the contention channel. If not, wait for an interval  $W_3$ , withdraw the carrier and try again later.
- Step 3 Wait until the current transmission is over and start transmitting the message, withdraw the carrier and exit.

Steps 1 and 2 are for collision avoidance and the protocol described is 1-persistent. Nonpersistent and other CSMA/CF variations can be found in [7]. We can see a clear distinction between this protocol and the CSMA/CD one. In CSMA/CF, the collision avoidance period is different from the transmission period while in CSMA/CD the detection period is overlapped with the transmission period.

Let  $a$  be the end to end propagation delay and  $t_0$  is the time of the first arrival. Any station that arrives in the interval  $[t_0, t_0+a+W_1]$  (called the vulnerable interval) will sense the channel idle and participates in the contention. Obviously, the condition that the first arrival wins the contention is that no other station arrives in this interval. On the other hand, if more than one stations participate in the contention, only the last arrival in the vulnerable contention has a chance of success. Let  $t_n, t_{n-1}$  be the last and the last but one arrival (if any) in that interval, the condition that the last arrival wins the contention is  $t_n - t_{n-1} \geq a + W_3$ . Figure 1 illustrates the CSMA/CF protocol (with  $t_0$  and  $t_n$  are the only two arrivals).

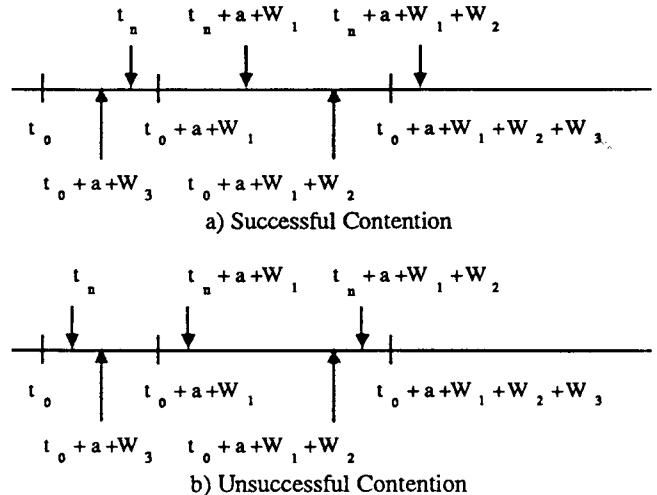


Figure 1 CSMA/CF Protocol

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In case a) of this figure, station  $n$  rechecks the contention channel at the time  $t_n + a + W_1 + W_2$  and finds out that station 0 already withdrew its carrier thus winning the contention. On the other hand, in case b) of the figure, station  $n$  can not detect that station 0 withdrew its carrier, and proceeds to withdraw its own carrier. This situation can be easily extended to many arrivals.

In Wong's earlier analysis of the protocol, throughput and delay were characterized. However, he assumed the system is heavily loaded in which every stations have messages to transmit. This make it hard to compare Wong's results with Tobagi's [10], and Takagi's [8] results on CSMA/CD studies which are based on Poisson traffic.

In this paper, a new analysis of the CSMA/CF protocol is presented. It is based on the following assumptions.

1. Newly arriving and backlogged messages are assumed to be generated from an infinite population by a Poisson process with a total arrival rate of  $\Lambda$ .
2. The steady state of the channel exists.
3. Propagation delay between any two stations is equal to a constant  $a$ .

Assumption 1 is widely used in analyses of random access protocols and has been proved to give reliable results [3]. Assumption 3 gives the worst case where the propagation delay between any two stations is the maximum propagation delay.

## II. THROUGHPUT ANALYSIS

The well known result from the renewal theory  $S = \frac{\bar{U}}{\bar{B} + \bar{I}}$  (where  $\bar{U}$ ,  $\bar{B}$  and  $\bar{I}$  are the expected value of the utilization, busy and idle periods) will be used to calculate the network throughput. Let us start by deriving the probability of success of a contention initiated by an arrival  $t_0$ . Let  $t_n, t_{n-1}$  be the last two arrivals (if any) in the vulnerable interval  $[t_0, t_0 + a + W_1]$ . If

$$y = t_n - t_0, \quad (1)$$

the probability distribution of  $y$  is

$$Y(t) = e^{-\Lambda(a+W_1 t)}$$

The density function of  $y$  follows

$$y(t) = \delta(t)e^{-\Lambda(a+W_1)} + \Lambda e^{-\Lambda(a+W_1 t)}$$

Let,  $P_s$  be the probability of success during a contention and  $P_s(y)$  be the probability of success given  $y$ . We can observe that if  $y = 0$ , (i.e. no arrival in the vulnerable interval) the probability of success is 1. If  $y < a + W_3$  the condition  $t_n - t_{n-1} \geq a + W_3$  could not be satisfied, i.e.  $P_s = 0, y < a + W_1$ . Otherwise, when  $y \geq a + W_3$  the probability of success is simply

$$P[t_n - t_{n-1} \geq a + W_3] = e^{-\Lambda(a+W_3)}$$

Then averaging over  $y$  we get the probability of success

$$P_s = e^{-\Lambda(a+W_1)} + \int_{a+W_3}^{\infty} e^{-\Lambda(a+W_1 y)} y(t) dt = e^{-\Lambda(a+W_3)}$$

Following the description of the CSMA/CF protocol, the transmission channel will alternate between idle and transmission periods. This idle period is a random variable which depends on the activities taking place on the contention channel. If a station arrives on an idle network (i.e. no transmissions or contentions are currently taking place), it starts a busy period. This busy period will be terminated with a transmission during which no messages contend for the network. The busy period starts with a contention period (called  $C_1$  with  $C_1(s)$  as the Laplace transform of its density function), at the end of which, the candidate for the next transmission is selected. Figure 2 shows the states of the channels.

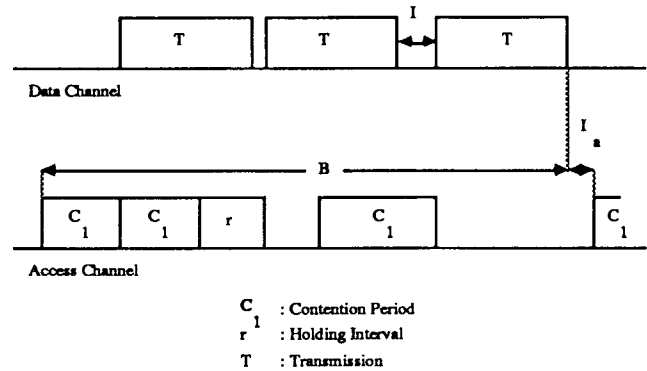
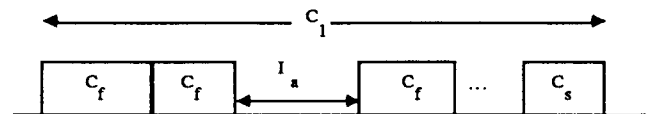


Figure 2 Busy period of 1-persistent CSMA/CF

The contention period consists of zero or more unsuccessful contention segments ( $C_f$ ), alternated with idle segments and terminated with a successful segment. The anatomy of a contention period is shown in figure 3.



$C_f$  : Unsuccessful Contention Segment

$C_s$  : Successful Contention Segment

$I_a$  : Idle Period on the Access Channel

$C_1$  : Contention Period

Figure 3 Contention period

Let us consider a contention period starting at time 0. If the first contention is successful, the duration of the contention period is simply the length of the contention,  $a + W_1 + W_2 + y$  with  $y$  defined earlier in (1). Otherwise, the length of the unsuccessful contention is  $a + W_1 + W_2 + W_3 + y$ .

During the first  $a + W_1$  seconds of the unsuccessful contention (i.e. the vulnerable interval), arriving stations will participate in the competition. For the remaining of the contention (i.e. the interval  $I_f = [a + W_1, a + W_1 + W_2 + W_3 + y]$ ), arriving messages will accumulate toward the next contention. If no messages accumulate, this contention segment will be followed by an idle

period ( $I_c$  with  $I_d(s)$  as the Laplace transform of its density function) and a subcontention period. This subcontention period will have the same probability distribution as the contention period itself. When exactly one message was accumulated during the last contention, the contention will be followed by a subcontention period with the same distribution. In the case that more than one message accumulates, the subsequent contention segment will have less chance of success. In such case, the probability of success is simply

$$P_2 = \int_{a+W_1}^{a+W_1+W_2} e^{-\Lambda(a+W_2)} y(t) dt = e^{-\Lambda(a+W_2)} e^{-\Lambda(a+W_1)}$$

Let  $C_2$  be the random duration of the subcontention period started by more than one station accumulated and  $C_2(s)$  be the Laplace transform of its density function we have

$$C_1(s) = e^{-s(a+W_1+W_2)} y(s) \{P_2 + P_f e^{-sW_2} [q_{1f} C_1(s) + q_{0f} I_d(s) C_1(s) + (1-q_{0f}-q_{1f}) C_2(s)]\} \quad (2)$$

where  $q_{0f}$ ,  $q_{1f}$  are the probabilities that no messages and exactly one message arrive during the interval  $I_f$ ,  $y(s)$  is the Laplace transform of  $y(t)$  and  $P_f = (1-P_2)$ .

The Laplace transform of the density function of the duration of the sub-contention period  $C_2$  can be found in a similar manner.

$$C_2(s) = e^{-s(a+W_1+W_2)} y(s) \{P_2 + P_f e^{-sW_2} [q_{1f} C_1(s) + q_{0f} I_d(s) C_1(s) + (1-q_{0f}-q_{1f}) C_2(s)]\} \quad (3)$$

where  $P_f^* = 1-P_2$ .

The system with two equations and two unknowns (2), (3) permits us to find  $C_1(s)$  and  $C_2(s)$ .

$$C_1(s) = [P_2 e^{-s(a+W_1+W_2)} y(s) + e^{-s(2a+2W_1+2W_2)} y^2(s) * (P_2 P_f - P_2 P_f^*) * (1-q_{0f}-q_{1f})] / [1 - e^{-s(a+W_1+W_2)} P_f (q_{1f} + q_{0f} I_d(s)) y(s) - e^{-s(a+W_1+W_2)} P_f^* (1-q_{0f}-q_{1f}) y(s)]$$

Depending on the number of messages accumulate at the end of the first contention period, the busy period will be followed by one of the three subbusy period types. Since the last contention of a contention period is a successful one, message only accumulate on the interval  $I_s = [a+W_1, a+W_1+W_2+y]$ . Let  $q_{0s}$ ,  $q_{1s}$  be the probability of no messages arrive and exactly one message arrives in  $I_s$ , we can write

$$\bar{B} = \bar{C}_1 + q_{1s} \bar{B}_1 + (1-q_{0s}-q_{1s}) \bar{B}_2 + q_{0s} q_{0T} \bar{T} + q_{0s} (1-q_{0T}) \bar{B}_0 \quad (4)$$

where

- $\bar{B}$ : the expected value of the busy period
- $\bar{B}_0$ : the expected value of the subbusy period starts with an idle period
- $\bar{B}_1$ : the expected value of the subbusy period generated by exactly one message accumulated

- $\bar{B}_2$ : the expected value of the subbusy period generated by more than one messages accumulated
- $T$ : the expected value of the transmission period
- $q_{0T}$ : the probability that no messages arrive during the transmission and
- $q_{1T}$ : the probability that exactly one message arrives during the transmission

To simplify the analysis, the message transmission time is assumed to be constant ( $T$ ).

#### a. The subbusy period starts with an idle period

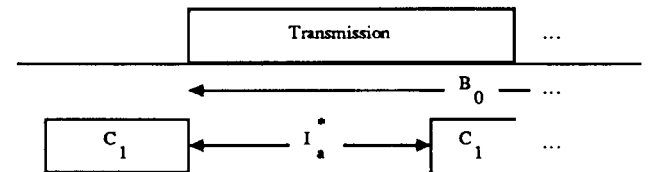
This subbusy period is illustrated in figure 4. If no messages arrive during the transmission, the subbusy period is simply the transmission period. Otherwise, an arrival will terminate the idle period ( $I_a^*$  with an expected duration of  $\bar{I}_a^*$ ) and generate a contention period ( $C_1$  with an expected duration of  $\bar{C}_1$ ). There are two possibilities, either  $I_a^* + C_1 > T$  or  $I_a^* + C_1 \leq T$ . When  $I_a^* + C_1 > T$  the subbusy period will be the sum of the duration of the idle period with duration of the contention and the duration of the sub-subbusy period which depends on the number of messages accumulated at the end of  $C_1$ . When  $I_a^* + C_1 \leq T$  the subbusy period is the sum of the transmission time with the duration of a sub-subbusy period (which depends on the number of message accumulated at the end of the transmission). Combining these two cases we can write

$$\bar{B}_0 = P[I_a^* + C_1 \leq T] (T + q_{1w} \bar{B}_1 + (1-q_{0w}-q_{1w}) \bar{B}_2 + q_{0w} (1-q_{0T}) \bar{B}_0 + q_{0w} q_{0T} T) + P[I_a^* + C_1 > T] (\bar{I}_a^* + \bar{C}_1 + q_{1s} \bar{B}_1 + (1-q_{0s}-q_{1s}) \bar{B}_2 + q_{0s} (1-q_{0T}) \bar{B}_0 + q_{0s} q_{0T} T) \quad (5)$$

where

$q_{0w}$ ,  $q_{1w}$ : the probabilities that no messages and exactly one message accumulated at the end of the transmission provided that  $I_a^* + C_1 \leq T$ .

$\bar{C}_1^{**}$ : the expected value of  $C_1$  provided that  $I_a^* + C_1 \leq T$ .



- $I_a^*$ : Idle Period given that it is shorter than the Transmission
- $C_1$ : Contention Period
- $B_0$ : Subbusy Period starts with an Idle Period

Figure 4 Subbusy period  $B_0$

#### b. The subbusy period generated by one message accumulated

An expression for  $\bar{B}_1$  can be derived in a similar way, noting that the subbusy period starts with a contention period ( $C_1$ ).

$$\begin{aligned} \bar{B}_1 = & P[C_1 \leq T] \{ T + q_{1w}^* \bar{B}_1 + (1 - q_{0w}^* - q_{1w}^*) \bar{B}_2 + q_{0s}^* (1 - q_{0T}) \bar{B}_0 + q_{0w}^* A \sigma T \} \\ & + P[C_1 > T] \{ \bar{C}_1^* + q_{1s} \bar{B}_1 + (1 - q_{0s} - q_{1s}) \bar{B}_2 + q_{0s} (1 - q_{0T}) \bar{B}_0 \\ & + q_{0s} A \sigma T \} \end{aligned} \quad (6)$$

where

$q_{0w}^*$ ,  $q_{1w}^*$ : the probabilities that no messages and exactly one message accumulated at the end of the transmission provided that  $C_1 \leq T$ .

$\bar{C}_1^*$ : the expected value of  $C_1$  provided that  $C_1 \leq T$ .

c. The subbusy period generated by more than one messages accumulated

The subbusy period generated by more than one message accumulated can be obtained by inspection, noting that it starts with  $C_2$  (defined earlier) instead of  $C_1$ .

$$\begin{aligned} \bar{B}_2 = & P[C_2 \leq T] \{ T + q_{1w}^{**} \bar{B}_1 + (1 - q_{0w}^{**} - q_{1w}^{**}) \bar{B}_2 + q_{0s}^{**} (1 - q_{0T}) \bar{B}_0 + q_{0w}^{**} A \sigma T \} \\ & + P[C_2 > T] \{ \bar{C}_2^* + q_{1s} \bar{B}_1 + (1 - q_{0s} - q_{1s}) \bar{B}_2 + q_{0s} (1 - q_{0T}) \bar{B}_0 \\ & + q_{0s} A \sigma T \} \end{aligned} \quad (7)$$

where

$q_{0w}^{**}$ ,  $q_{1w}^{**}$ : the probabilities that no messages and exactly one message accumulated at the end of the transmission provided that  $C_2 \leq T$ .

$\bar{C}_2^*$ : the expected value of  $C_2$  provided that  $C_1 \leq T$ .

From equations (4)-(7), the expected value of the busy period can be calculated. Expressions for several parameters can be found in the appendix. The only remaining term needed to calculate the throughput is the expected length of the utilization period. If no messages accumulated during the contention and no messages arrive during the transmission, the transmission period is simply  $T$ . Otherwise, it is the sum of the transmission with the subutilization period, which in turn has the same distribution. We can write

$$\bar{U} = T + (1 - q_{0s} A \sigma T) \bar{U}$$

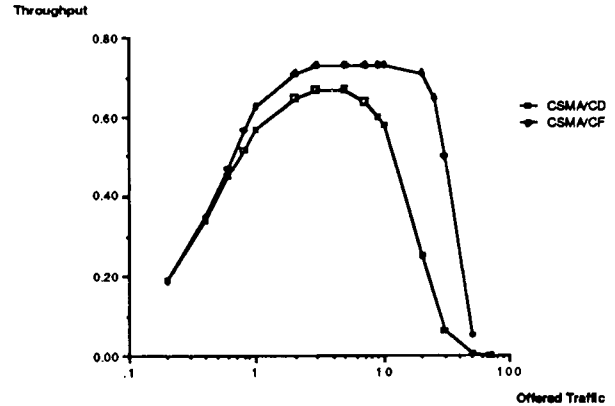
$$\text{which gives } \bar{U} = \frac{T}{q_{0s} A \sigma T}$$

$$\text{The throughput is simply } S = \frac{\bar{U}}{\bar{B} + 1/\Lambda}$$

### III. NUMERICAL RESULTS

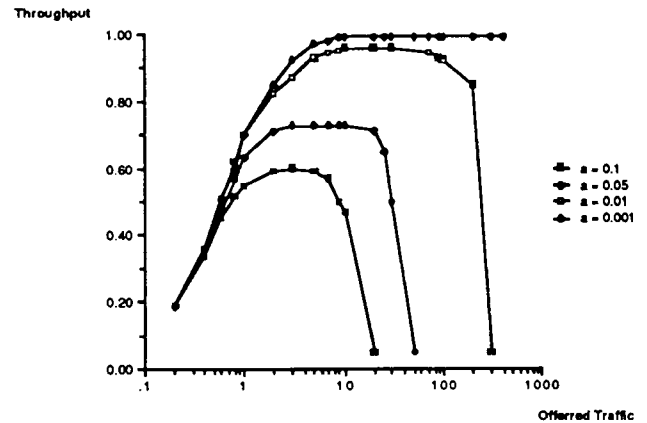
Figure 5 shows a comparison between CSMA/CD and CSMA/CF throughput. The throughput of the collision free protocol is greatly increased, mostly due to the employment of prescheduling and parallelism. On the graph we can see that CSMA/CF throughput also decreases at heavy load however it

offers more stability than CSMA/CD. Figure 6 shows CSMA/CF throughput for different sets of parameters. As the propagation delay tends to zero, the throughput persists longer. Since the propagation delay in local area network is generally small, CSMA/CF protocol is best suited for them. The protocol has been proposed to be used in the Homogeneous Multiprocessor system [1] as an interprocessor communication access method. A VLSI chip supporting the CSMA/CF protocol also has been designed [6].



$$a=0.05, W_1=0.05, W_2=0.15, W_3=0, T=1, d=0.15$$

Figure 5 Throughput of CSMA/CF vs CSMA/CD



$$W_1 = a, W_2 = 3a, W_3 = 0$$

Figure 6 Throughput of CSMA/CF for different propagation delays

### IV. CONCLUSIONS

An analysis of the 1-persistent CSMA/CD has been presented. The new analysis is based on the same set of assumptions exploited by most of existing analyses on the multiple access methods, which provides a more acceptable basis for comparing CSMA/CD and CSMA/CF protocols. Delay analysis is also possible by adopting the Markov chain approach employed by

Lam [4]. Equations (4)-(7) can be written in forms of Laplace transform to facilitate the analysis.

## VI. APPENDIX

First, the average idle period given that the idle period is shorter than the transmission time is

$$\bar{I}_a^* = \int_0^T \frac{t \Lambda e^{-\Lambda t}}{1 - e^{-\Lambda T}} dt = \frac{(1 - e^{-\Lambda T}) - \Lambda T e^{-\Lambda T}}{\Lambda(1 - e^{-\Lambda T})}$$

Various probabilities and terms in equations (4)-(7) can be found using the numerical Laplace transform inversion.

$$P[C_1 \leq T] = \int_0^T C_1(t) dt = \mathcal{L}^{-1} \{ C_1(s)/s \} \Big|_{t=T}$$

$$P[C_2 \leq T] = \int_0^T C_2(t) dt = \mathcal{L}^{-1} \{ C_2(s)/s \} \Big|_{t=T}$$

$$P[I_a^* + C_2 \leq T] = \mathcal{L}^{-1} \{ I_a^*(s) C_2(s)/s \} \Big|_{t=T}$$

$$P[C_1 > T] \bar{C}_1^* = \int_T^\infty C_1(t) dt = \int_0^\infty C_1(t) dt - \int_0^T C_1(t) dt \\ = \bar{C}_1 - \mathcal{L}^{-1} \{ -C_1'(s)/s \} \Big|_{t=T}$$

$$P[C_2 > T] \bar{C}_2^* = \int_T^\infty C_2(t) dt = \int_0^\infty C_2(t) dt - \int_0^T C_2(t) dt \\ = \bar{C}_2 - \mathcal{L}^{-1} \{ -C_2'(s)/s \} \Big|_{t=T}$$

using the property

$$\mathcal{L} \{ t f(t) \} = -F'(s)$$

where  $F'(s)$  denotes the first derivative  $dF(s)/ds$ .

and

$$P[I_a^* + C_1 > T] (\bar{I}_a^* + \bar{C}_1^*) = \bar{I}_a^* + \bar{C}_1 - \mathcal{L}^{-1} \left\{ -\frac{1}{s} \frac{d}{ds} [I_a^*(s) C_1(s)] \right\} \Big|_{t=T}$$

with

$$I_a^*(s) = \frac{\Lambda [1 - e^{-(\Lambda+s)T}]}{(\Lambda+s)(1 - e^{-\Lambda T})} \text{ from } I_a^*(t) = \frac{\Lambda e^{-\Lambda t}}{1 - e^{-\Lambda T}}, 0 \leq t \leq T.$$

As defined earlier,  $q_{0w}^*$  is the probability that no messages accumulate toward the end of the transmission, given that  $C_1 < T$ .

Now, if we define  $C^*$  the contention length excluding the last  $W_2 + y$  seconds (during which arrivals find the contention channel busy and start accumulate), the Laplace transform of its density function is

$$C^*(s) = \frac{C_1(s)}{e^{-sW_2} y(s)}$$

The generating function of the number of messages accumulating toward the end of the transmission is simply the generating function of the number of arriving messages find the contention channel busy.

$$W^*(z) = \frac{e^{-sT}}{C^*(s)} \Big|_{s=\Lambda(1-z)}$$

$$q_{0w}^* = W^*(0) \text{ since } W^*(z) = \sum_{i=0}^{\infty} q_{iw}^* z^i$$

$$q_{1w}^* = \int_0^T C^*(t) \Lambda(T-t) e^{-\Lambda(T-t)} dt = \mathcal{L}^{-1} \left\{ C^*(s) \frac{\Lambda}{(s+\Lambda)} \right\} \Big|_{t=T}$$

Similarly,

$$q_{0w}^{**} = W^{**}(0)$$

with

$$W^{**}(z) = \frac{e^{-sT}}{C^{**}(s)} \Big|_{s=\Lambda(1-z)} \text{ and } C^{**}(s) = \frac{C_2(s)}{y(s) e^{-sW_2}}$$

and

$$q_{1w}^{**} = \int_0^T C^{**}(t) \Lambda(T-t) e^{-\Lambda(T-t)} dt = \mathcal{L}^{-1} \left\{ C^{**}(s) \frac{\Lambda}{(s+\Lambda)} \right\} \Big|_{t=T}$$

Finally,

$$q_{0w} = W(0)$$

$$q_{1w} = \mathcal{L}^{-1} \left\{ C(s) \frac{\Lambda}{(s+\Lambda)} \right\} \Big|_{t=T}$$

where

$$W(z) = \frac{e^{-sT}}{C(s)} \Big|_{s=\Lambda(1-z)} \text{ and } C(s) = \frac{I_a^*(s) C_1(s)}{y(s) e^{-sW_2}}$$

As mentioned earlier, we define  $q_{0s}$  is the probability of no arrival in the interval  $l_s$  given that the contention is successful.

$$q_{0s} = \frac{e^{-\Lambda(a+W_2)} e^{-\Lambda W_2}}{e^{-\Lambda(a+W_2)}} + \frac{1}{e^{-\Lambda(a+W_2)}} \int_{a+W_2}^{a+W_2+y} e^{-\Lambda(a+W_2)} \Lambda e^{-\Lambda(a+W_2+t)} e^{-\Lambda(W_2+t)} dt \\ = e^{-\Lambda(W_2+W_2)} + \Lambda (W_2 - W_3) e^{-\Lambda(W_2+W_2+a)}$$

and

$$q_{1s} = \frac{e^{-\Lambda(a+W_2)} \Lambda W_2 e^{-\Lambda W_2}}{e^{-\Lambda(a+W_2)}} \\ + \frac{1}{e^{-\Lambda(a+W_2)}} \int_{a+W_2}^{a+W_2+y} e^{-\Lambda(a+W_2)} \Lambda e^{-\Lambda(a+W_2+t)} \Lambda (W_2+t) e^{-\Lambda(W_2+t)} dt$$

$$= \Lambda W_2 e^{-\Lambda(W_1+W_2+W_3)} + e^{-\Lambda(a+W_1+W_2)} \Lambda^2 \left( W_2(W_1-W_3) + \frac{(a+W_1)^2 - (a+W_3)^2}{2} \right)$$

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$q_{0f}$ ,  $q_{1f}$  can be found in a similar manner.

The only remaining terms are  $q_{0T}$ ,  $q_{1T}$ . Since  $T$  is a constant, finding this two terms is trivial.