# Collision-free protocol for local area networks

Nikitas J Dimopoulos\* and Eric Chi-Wah Wong<sup>†</sup> propose a scheme which eliminates collisions in a network

This paper defines a new Carrier Sense Multiple Access (CSMA) protocol, where the activities of channel contention and packet transmission occur concurrently. This is achieved through the introduction of a contention channel which is independent of the main transmission channel. The new protocol is proven correct, in the sense that it guarantees collision-free transmission of packets. Its performance is also evaluated and compared with that of a CSMA/CD protocol.

Keywords: local area networks, protocols, CSMA, collision-free transmission, contention channel, transmission channel

A Local Area Network (LAN) is a communications network that provides interconnection of a variety of datacommunicating devices within a small area. LANs employ various topologies, transmission media and channel access protocols. Channel access protocols can be categorized as fixed, contention and demand assignment protocols. As contention protocols we characterize protocols such as ALOHA<sup>1</sup>, Carrier Sense Multiple Access (CSMA) and Carrier Sense Multiple Access with Collision Detection (CSMA/CD) protocols<sup>2, 3</sup>. CSMA/CD protocols have been employed in several LANs, the most prominent one being Ethernet<sup>4</sup>.

A CSMA/CD protocol requires that each ready station senses the channel prior to transmitting its own packet, and will never initiate a transmission when it senses that the channel is busy. If a ready station senses that the

\*Electrical and Computer Engineering Department, University of Victoria, PO Box 1700, Victoria, British Columbia, Canada V8W 2Y2 <sup>†</sup> CANSTAR Inc., Toronto, Ontario, Canada channel is idle, it initiates a packet transmission, and at the same time it checks for collisions. If a collision is detected, the current packet transmission is aborted. In a CSMA/CD protocol collisions do exist, and in heavy traffic contribute to the deterioration of the channel utilization.

A scheme is proposed here which eliminates the existence of collisions in a network. In such a scheme, pre-scheduling and parallelism are incorporated, i.e. in the proposed protocol, the ready stations contend for the network during the transmission of a packet. Thus, at the end of the current transmission period, it is expected (with a high probability) that the next master of the network has been chosen. Therefore, collisions are eliminated, and the utilization of the network increases correspondingly.

In such a scheme the two activities of channel contention and packet transmission must occur without interference from each other, therefore they are provided with two separate channels. The contention channel is used by the stations to decide on the next master of the network, while the transmission channel is used for the actual packet transmissions. The contention channel is normally of a much lower capacity than the transmission channel. In this implementation<sup>5, 6</sup> the transmission channel is a 16-line parallel bus, while the contention channel utilizes a single line.

Similar approaches have been proposed by Hamacher et  $al.^7$ , with a contention channel resembling a token ring rather than a bus, by Mark<sup>8</sup>, with a slotted contention channel, and by Jafari et  $al.^9$ , where the contention channel is a loop, with a loop master that controls traffic on the 'contention loop' and access to the 'data loop'. In this scheme the contention channel has the topology of a bus, which does not impose any transmission priorities on the competing stations, and also avoids the time delays incurred during the passing of the token.

0140-3664/88/040208-07 \$03.00 © 1988 Butterworth & Co (Publishers) Ltd

# COLLISION-FREE PROTOCOL: THE PEBBLE PARADIGM

In order to explain the new Collision-Free (CSMA/CF) protocol the following paradigm is used: the transmission channel is considered as a resource to be shared among several stations; the contention channel plays a similar role as a semaphore<sup>10</sup>, and its function is to ensure that at most one station will be allocated to the transmission channel at any particular time.

Each station is capable of transmitting a carrier signal over the contention channel, and is also capable of distinguishing whether zero, one or more carrier signals are present (an example of such a channel, and the hardware required, may be found in Reference 11).

The contention channel may be thought of as a vessel containing zero or more pebbles (a pebble in this context corresponds to a single carrier signal transmitted over the contention channel). The vessel is in public view, and each ready station may inspect it to determine the number of pebbles (carrier signals) present. A ready station is also capable of depositing a pebble into the vessel, as well as of retrieving a single pebble (corresponding to the initiation or termination of the transmission of its own carrier). In addition, a network station is capable of determining whether the transmission channel is free or not. All of these actions take a finite time, in accordance with the signal propagation and processing delays in the network.

A network station is ready if it needs to transmit a packet over the transmission channel, and it follows the following protocol: at time  $t_0$  a ready station inspects the vessel with the pebbles; if the vessel contains at least one pebble (indicating that a contention is already in progress), the attempting station is blocked, and it must try again at a later time; otherwise, if the vessel is found to be empty, then the station deposits its pebble at time  $t_0 + w_1$ .

The delay  $w_1$  was introduced here because two different operations (inspecting the vessel and depositing the pebble) are performed by the attempting station. Because of the finite processing speed, these two operations can neither happen instantaneously nor simultaneously.

Once a station has deposited its pebble, thus obtaining the right to compete for the use of the transmission channel, it must ensure that it is the only one with this right. It therefore waits until all the other ready stations have deposited their pebbles in the vessel, and then re-examines the vessel at time  $t_0 + w_1 + w_2$ . The time delay  $w_2$  is chosen so that all potential ready stations are given the opportunity to deposit their pebbles in the vessel, and also it reflects the various signal propagation delays over the network (the range of values for the delay  $w_2$  is calculated below).

If at time  $t_0 + w_1 + w_2$  the station finds only one pebble in the vessel, then it is assured that it is the only station remaining, and can therefore utilize the transmission channel as soon as the current transmission period terminates. Such a station is called a 'successful' station.

A successful station withdraws its pebble simultaneously with the start of the transmission of its own packet, so that the start of the next contention period coincides with the start of a packet transmission period. Otherwise, if at time  $t_0 + w_1 + w_2$  the station finds more than one pebble in the vessel, then it withdraws its pebble at time  $t_0 + w_1 + w_2 + w_3$ , and re-executes the protocol. Again, the introduction of the delay  $w_3$  is due to the two distinct operations of determining the number of pebbles in the vessel, and of withdrawing a single pebble.

The values of the delays  $w_1$ ,  $w_2$  and  $w_3$  depend on the implementation as well as on each other. These interdependencies are derived below, where the correctness of the CSMA/CF protocol stated above is also considered. Suffice to say that the protocol operates correctly if  $w_2 > w_1 + 2a$  and  $w_3 < w_1$ .

### CSMA/CF PROTOCOL PERFORMANCE ANALYSIS

### Conditions for successful transmission

There are two possibilities under which an arriving station will be allowed, at the end of a contention period, to become successful (i.e. to become eligible for the acquisition of the transmission channel for the next transmission period):

- case 1 the arriving station is the only ready station.
- case 2 the arriving station competes with other arriving stations and successfully completes the current contention period (i.e. it finds only one pebble in the vessel when it re-examines it at time t<sup>5</sup>).

For case 1, the arriving station  $s_i$  will become successful if no other ready station arrives within its blocking period, i.e. there are no arrivals within the interval  $[t_i, t_i + w_1^i + a]$ (the corresponding probability of occurrence of this case is calculated below).

For case 2, where a number of ready stations compete, the station which finds the vessel containing only one pebble will become successful, and will use the transmission channel during the next transmission period. For the second case, we proceed as follows: suppose that there are k competing stations (out of a population of N ready stations); k = 1, 2, ..., N - 1. As is evident, the k stations arrive within the blocking interval of the first arriving station  $s_0$ , while the remaining N - k stations arrive later on, find the vessel contains more than one pebble, and are blocked (see Figure 1).

Each of the k stations arrive at times  $t_i$ ; i = 1, 2, ..., k, and deposit their pebbles at times  $t'_1 = t_i + w'_1$ . All the stations check the vessel at times  $t'_2 = t_i + w'_1 + w'_2$ , and if they find more than one pebble then they withdraw their own pebble at time  $t'_3 = t_i + w'_1 + w'_2 + w'_3$ .

Sufficient conditions are now shown on the time delays  $w_3^i$  so that the case of continuous collision is avoided. Of course, a collision may still happen for a certain mix of arrival times, the probabilities of which are calculated below.

In order for a station  $s_j$  to become successful, it must, when it re-examines the vessel at time  $t_2^j$ , find that it contains one pebble, i.e. all the other stations must have



Figure 1. Station arrivals for case 2a

withdrawn their own pebbles before  $t_2^i$ . Otherwise, a collision is detected and station  $s_j$  withdraws its own pebble at time  $t_3^i$ .

In order for all the stations to collide,  $w_3^i$  must be chosen so that when all the stations check the channel at time  $t_2^i$ , they find that it contains more than one pebble. This is guaranteed to happen if the latest arriving station still finds the pebble of the first arriving station present in the vessel, or, taking into account the propagation delays, we have for the case of continuous collisions max  $\{t_2^i\} < \min\{t_3^i + a\}$  since:

$$\max_{i} \{t_{2}^{i}\} = t_{BLOCK} + \max_{i} \{w_{1}^{i}\} + \max_{i} \{w_{2}^{i}\} = t_{0}$$

$$+ 2 \max_{i} \{w_{1}^{i}\} + \max_{i} \{w_{2}^{i}\}$$
and 
$$\min_{i} \{t_{3}^{i}\} = t_{0} + \min_{i} \{w_{1}^{i}\} + \min_{i} \{w_{2}^{i}\}$$

$$+ \min_{i} \{w_{3}^{i}\} + a$$
then 
$$\min_{i} \{w_{3}^{i}\} > 2 \max_{i} \{w_{1}^{i}\} - \min_{i} \{w_{1}^{i}\} + \max_{i} \{w_{2}^{i}\}$$

$$- \min_{i} \{w_{1}^{i}\}$$

Thus, if we choose:

$$w_3^i < 2 \max_i \{w_1^i\} - \min_i \{w_1^i\} + \max_i \{w_2^i\} - \min_i \{w_2^i\}$$
(1)

we avoid the situation where continuous collisions happen when the stations compete for the contention channel.

On the other hand, for a station  $s_j$  to become successful, its arrival time must satisfy:

$$t_2^i > t_3^i + a; i \neq j$$
 (2)

In order to simplify the calculations, for the remainder of the paper the waiting times are considered to be identical for all the stations involved, i.e.  $w_1^i = w_1^i$ ,  $w_2^i = w_2^i$ ,  $w_3^i = w_3^i$ ;  $i, j = 1, 2, ..., N_i$ . This is a reasonable assumption, since all the network stations are identical to each other. Under this assumption, conditions (A2) (see Appendix) and (1) become:

$$w_2 > w_1 + 2a \tag{A2a}$$

$$w_3 < w_1 \tag{1a}$$

Under the same assumption, times  $t_1^i$ ,  $t_2^i$  and  $t_3^i$  are ordered in the same way as their corresponding arrival times  $t_i$ . Therefore, the only station which can potentially acquire the network is the last arriving station before the blocking time  $t_{BLOCK}$ . Thus, equation (2) yields:

$$t_i - t_{i-1} > w_3 + a$$
 (2a)

## Success probability in obtaining the network after one contention

In addition to the assumptions outlined earlier, it is also assumed that there are always N competing stations, and that the elapsing time between successive attempts by a station to gain the network is constant and finite. This period is denoted as T.

Thus, the probability distribution of the arrival time of a

station attempting to acquire the network is continuous and uniformly distributed in [0, T].

Two cases are considered: for the first there are no arrivals within the blocking interval  $[t_0, t_{BLOCK}]$ , i.e. there is only one station arriving at time  $t_0 \in [0, T]$ , and this station obtains the network at time  $t_0 + w_1 + w_2$ ; for the second case there are several stations arriving within the blocking interval  $[t_0, t_{BLOCK}]$ , with the last one arriving at time  $t_i \leq t_{BLOCK}$  given the chance to acquire the network.

#### Case 1

The probability of success for this case is calculated as follows:

$$dP_1(t) = \mathbf{Pr} \{ \text{the first arrival within } [t, t + dt] \text{ and no} \\ arrivals within the blocking interval} \\ [t, t + w_1 + a]; t \in [0, T] \}$$

Since the arrival time for each station is uniformly distributed in [0, T], and as the stations are independent of each other, we have:

$$dP_{1} = N\left(\frac{dt}{T}\right) \left[H\left(\frac{T-t-w_{1}-a}{T}\right)\right]^{N-1}$$
(3)  
$$H(x) = \begin{cases} 0 \text{ if } x < 0\\ x \text{ if } x > 0 \end{cases}$$

Integrating (3) we obtain:

$$P_{1} = \int_{0}^{T} dP_{1}(t) = \left(\frac{T - w_{1} - a}{T}\right)^{N}$$
(4)

where N is the number of stations competing for the network.

#### Case 2

As mentioned earlier, this case involves at least one arrival within the blocking interval  $[t_0, t_{BLOCK}]$ . For this case, the important calculation times are the arrival times of the first and last stations ( $s_0$  and  $s_i$  respectively). Two possibilities can be distinguished, depending on the arrival time of the first station  $s_0$ .

#### Case 2a: $0 < t_0 < T - w_1 - a$

Under this assumption, the blocking interval  $[t_0, t_{BLOCK}]$ lies entirely within the latency interval [0, T]. As shown in Figure 1, there are i - 1 arrivals in the interval  $[t_0, t_i - w_3 - a]$  (refer to relations (2) and (2a) above), while the remaining N - i - 2 arrivals happen after  $t_{BLOCK}$ .

Thus, the probability of success for the last arriving station (station  $s_i$ ) is given as:

$$dP_{2}^{*}(\xi, t) = \mathbf{Pr} \{ \text{the first arrival within } [\xi, \xi + d\xi), \\ \text{one arrival within } [t, t + dt); \\ \text{no arrivals within } [t - w_{3} - a, t] \\ \text{and } (t + dt, t_{BLOCK}]; \\ t_{BLOCK} > t > \xi + d\xi + w_{3} + a; \\ t_{BLOCK} = \xi + w_{1} + a \text{ and } t \in [0, T], \\ \xi \in [0, T - w_{1} - a] \}$$
(5)

Referring to Figure 1, and taking into account all the possible distributions of the N-2 arrival times in the intervals [ $\xi$ ,  $t - w_3 - a$ ] and [ $t_{BLOCK}$ , T], we obtain from equation (5):

$$dP_{2}^{*}(\xi) = N\left(\frac{d\xi}{T}\right) \int_{\xi+w_{3}+a}^{t_{BLOCK}} (N-1)\frac{dt}{T} \sum_{i=0}^{N-2} \binom{N-2}{i}$$
$$\left(\frac{t-w_{3}-a-\xi}{T}\right)^{i} \binom{N-i-2}{N-i-2}$$
$$\left(\frac{T-t_{BLOCK}}{T}\right)^{N-i-2}$$
$$= \frac{N!}{T} \left(\frac{d\xi}{T}\right) \sum_{i=0}^{N-2} \frac{T}{(N-2-i)!(i+1)!}$$
$$\left(\frac{w_{1}-w_{3}}{T}\right)^{i+1} \left(\frac{T-t_{BLOCK}}{T}\right)^{N-i-2}$$
(6)

Equation (6) gives the probability of success distribution in terms of the arrival time  $\xi$  of the first arriving station. Thus, the probability of success can be obtained through the integration of equation (6) over the domain of  $\xi$ , i.e.:

$$P_{2}^{*} = \int_{0}^{T - w_{1} - a} dP_{2}^{*}(\xi)$$
$$= \frac{1}{T^{N}} \left[ (t - w_{3} - a)^{N} - (T - w_{1} - a)^{N} - (w_{1} - w_{3})^{N} \right]$$
(7)

Case 2b:  $T - w_1 - a < t_0 < T$ 

For this case there is still more than one station arriving within the blocking interval  $[t_{Q}, t_{BLOCK}]$ , but the first arriving station arrived late, so that the blocking time extends beyond the latency time, i.e.  $t_{BLOCK} > T$  (see Figure 2). Then, the probability of success (again of the last arriving station  $s_{N-1}$ ) is given as:

$$\begin{aligned} dP_2'(\xi,t) &= \mathbf{Pr} \{ \text{the first arrival within } [\xi, \xi + d\xi), \text{ one} \\ & \text{arrival within } [t, t + dt); \text{ no arrivals} \\ & \text{within } [t - w_3 - a, t]; \ T > t > \xi + d\xi \\ & + w_3 + a; \ t \in [0, T]; \ \xi \in [T - w_1 - a, \\ T] \} \end{aligned}$$

As this is a successful transmission, according to relation (2a), all the arriving stations must arrive at least  $w_3 + a$  time units before the last one  $(s_{N-1})$ . Thus, there are N-2 arrivals in the interval ( $\xi$ ,  $t - w_3 - a$ ], and the probability of success is calculated as:

$$\partial F_2(\zeta) = N\left(\frac{d\xi}{T}\right)(N-1)\int_{\xi+w_3+a}^T \left(\frac{T-w_3-a-\xi}{T}\right)^{N-2}\frac{dt}{T}$$



Figure 2. Station arrivals for case 2b

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$$dP'_{2}(\xi) = N\left(\frac{d\xi}{T}\right)\left(\frac{t-w_{3}-a-\xi}{T}\right)^{N-1}$$
(8)

Integrating equation (8) for all the possible values of  $\xi$ , i.e. the arrival time of the first station, we obtain:

$$P'_{2} = \frac{N}{T} \int_{T-w_{1}-a}^{T-w_{3}-a} \left(\frac{T-w_{3}-a-\xi}{T}\right)^{N-1} d\xi$$
$$P'_{2} = \left(\frac{w_{1}-w_{3}}{T}\right)^{N}$$
(9)

Combining equations (4), (7) and (9) the probability of success can be calculated as:

$$P_{s} = P_{1} + P_{2}^{*} + P_{2}^{\prime} = \left(\frac{T - w_{1} - a}{T}\right)^{N}$$
  
+  $\frac{1}{T^{N}} \left[ (T - w_{3} - a)^{N} - (T - w_{1} - a)^{N} - (w_{1} - w_{3})^{N} \right] + \left(\frac{w_{1} - w_{3}}{T}\right)^{N}$   
=  $\left(\frac{T - w_{3} - a}{T}\right)^{N}$  (10)

Similarly, the probability of failure for a single station to acquire the network after one contention period is given as:

$$P_{f} = 1 - P_{s} = 1 - \left(\frac{T - w_{3} - a}{T}\right)^{N}$$
 (11)

The probability of success is now used to calculate the average idle time for a network operating under the new CSMA/CF protocol (recall that as there are no collisions in this scheme, the average transmission and busy periods are equal).

# Average idle period and utilization factor calculation

As there are separate contention and transmission channels in this scheme, the network stations may contend while a packet is transmitted over the transmission channel. The probability of failure, as given by equation (11), refers to a single contention period. It becomes evident that as the number of stations Nincreases, the probability of failure approaches 1. Thus, in very heavy traffic we expect that the number of contentions required before the next master is decided will increase. As an effect of this there is a corresponding increase of the total contention period. When the total contention period surpasses the current packet transmission period, idling of the network occurs, with a corresponding decrease in the network utilization factor.

This section aims to provide the calculation for the average idle period as it depends on the number of stations contending. The final goal is to compute the average utilization factor, as well as the average packet delay incurred in a network operating under the new CSMA/CF protocol. The activities on the transmission and contention channels are depicted in Figure 3.

Denote by x the length of an unsuccessful contention period, by y the length of a successful contention period,



Figure 3. Emergence of an idle period. X: station gains mastership of network for the next transmission period and waits for the transmission channel to become idle; +: transmitting station has completed its transmission and releases the transmission channel — the waiting station gets the channel and starts to transmit; O: there are many unsuccessful contention periods — the total contention interval is longer than the transmission period, therefore there is an idle period

and by  $C_k$  the length of a total contention period, composed of k - 1 unsuccessful plus one successful contention. These quantities are then related as:

$$C_k = (k-1)x + y$$
 (12)

and  $x = t + w_1 + w_2 + w_3 + a$ ,  $y = t + w_1 + w_2$  where t, being the arrival time of the last station, is a random variable distributed within the interval  $[t_0, t_{BLOCK}]$ . In order to simplify the mathematics, the upper bound of the contention interval is calculated. This is obtained when the arrival time of the last station coincides with  $t_{BLOCK}$ . Thus,  $t = t_{BLOCK} = t_0 + w_1 + a$ . Since  $t_0$  is the arrival time of the first (of N) arriving stations, and  $w_1$  and a are constants, the mean and standard deviation of t are given as:

$$\overline{t} = \frac{T}{N+1} + w_1 + a \text{ and } \sigma^2 = T^2 \frac{N}{(N+2)(N+1)^2}$$

The length of an idle period is therefore given as  $I_k = \mathbf{H}(C_k - \tau)$ , where  $\tau$  is the packet transmission period and k is the number of contention periods.

The average idle period  $\overline{l}$  can then be obtained as  $\overline{l} = \mathbb{E}[\overline{l_k}]$ , where  $\overline{l_k}$  is the expected value of an idle period obtained from k contentions. These two quantities can be calculated as follows:

$$\overline{I} = \mathbb{E}[\overline{I}_{k}] = \sum_{k=1}^{\infty} \Pr[\text{there are } k \text{ contention periods}]\overline{I}_{k}$$
$$= \sum_{k=1}^{\infty} \Pr_{I}^{k-1} \Pr_{S} \overline{I}_{k} \qquad \text{ contention periods}] (13)$$

<sup>†</sup> Given N arriving stations with arrival times t, uniformly distributed in the interval [0, T], then the density function f(t) of the first arrival  $t = \min_r(t_r)$  is calculated as:

$$dP(t_0) = f(t_0)dt = Pr[N - 1 arrivals after t_0 plus one arrival in dt_0]$$

 $= N \left( \frac{T - t_0}{T} \right)^{N - 1} \frac{dt_0}{T}$ 

Thus, the expected value of the arrival time of the first arriving station, is given as:

$$\overline{t_0} = \int_0^T t_0 dP(t_0) = \int_0^T t_0 N \left(\frac{T - t_0}{T}\right)^{N-1} \frac{dt_0}{T} = \frac{T}{N+1}$$

while the standard deviation is obtained as:

$$\sigma^{2} = \int_{0}^{T} \left( t_{0} - \frac{T}{N+1} \right)^{2} N \left( \frac{T-t_{0}}{T} \right)^{N-1} \frac{dt_{0}}{T} = T^{2} \frac{N}{(N+2)(N+1)^{2}}$$

and 
$$\overline{I_k} = \int_{\tau}^{\infty} (z - \tau) P(C_k = z) dz$$
 (14)

In order to evaluate equations (13) and (14),  $P(C_k = z)$  should be calculated. By means of the central limit theorem<sup>12</sup>,  $P(C_k = z)$  can be approximated as:

$$P(C_k = z) = \frac{1}{\sigma_k \sqrt{2\pi}} e^{-(z - \eta_k)^2 / 2\sigma_k^2}$$
(15)

where  $\eta_k = (k-1)\eta_x + \eta_y$  and  $\sigma_k^2 = (k-1)\sigma_x^2 + \sigma_y^2$  (16)

$$\eta_x = E[x] = \frac{T}{N+1} + 2w_1 + w_2 + w_3 + 2a.$$
 (17)

$$\eta_y = E[y] = \frac{T}{N+1} + 2w_1 + w_2 + a \tag{18}$$

$$\sigma_x^2 = \sigma_y^2 = T^2 \frac{N}{(N+2)(N+1)^2}.$$
 (19)

Combining equations (16), (17), (18) and (19) we obtain:

$$\eta_{k} = k \frac{T}{N+1} + k(2w_{1} + w_{2} + a) + (k-1)(w_{3} + a)$$
(20)

$$\sigma_k^2 = T^2 \frac{N}{(N+2)(N+1)^2}$$
(21)

The average idle period given k contentions is calculated by using equations (14) and (15) as:

$$\overline{I_k} = \int_{\tau}^{\infty} (z - \tau) \frac{1}{\sigma_k \sqrt{2\pi}} e^{-(z - \eta_k)^2 / 2\sigma_k^2 dz} = \frac{\sigma_k}{\sqrt{2\pi}} e^{-(\tau - \eta_k)^2 / 2\sigma_k^2} + \frac{\eta_k - \tau}{2} \operatorname{erf}\left(\frac{\tau - \eta_k}{\sqrt{2\sigma_k}}\right)$$
(22)

where **erf** is defined as  $\mathbf{erf}(\mathbf{a}) = \frac{2}{\sqrt{\pi}} \int_{\mathbf{a}}^{\infty} \mathbf{e}^{-t^2} dt$ . It has

been proven<sup>11</sup> that the series given in equation (13) converges.

Equation (13) has been used to numerically compute the average utilization factor of a network operating under the new CSMA/CF protocol. The average utilization factor  $\overline{S}$  is given by the well known<sup>2</sup> formula:

$$\overline{S} = \frac{\overline{U}}{\overline{I} + \overline{B}}$$
(23)

In this case, as there are no collisions,  $\overline{U} = \overline{B} = \overline{\tau}$ , and equation (23) can be simplified into:

$$\overline{S} = \frac{\overline{\tau}}{\overline{1 + \overline{\tau}}}$$
(24)

As it has been considered that there are always N active stations on the network, each one of which attempts to transmit one packet, then by applying Little's result<sup>13</sup>, the average packet delay  $\overline{D}$ , as a function of the number of stations N, can be evaluated as:

$$\overline{D} = \frac{N}{\overline{5}}$$
(25)

#### DISCUSSION

Equations (24) and (25) were used in order to evaluate the average utilization factor and the average packet delay as functions of the number of nodes N involved, and for the characteristic waiting times. These functions are depicted in Figures 4 and 5.

The corresponding utilization factor for both a 1-persistent CSMA/CD<sup>14</sup> and the new CSMA/CF protocols with similar channel characteristics have also been incorporated in the same plot (see Figure 6). The advantage of the new CSMA/CF protocol over the classical CSMA/CD becomes clear in this figure.



Figure 4. Average utilization factor as a function of the number of nodes for the CSMA/CF protocol.  $\blacksquare$ : case 1 —  $\tau = 15.0$ , T = 1.0, a = 0.1,  $w_1 = 0.1$ ,  $w_2 = 0.3$ ,  $w_3 = 0.0$ ; +: case 2 —  $\tau = 30.0$ , T = 1.0, a = 0.1,  $w_1 = 0.1$ ,  $w_2 = 0.2$ ,  $w_3 = 0.0$ ;  $\Box$ : case 3 —  $\tau = 60.0$ , T = 1.0, a = 0.1,  $w_1 = 0.1$ ,  $w_2 = 0.3$ ,  $w_3 = 0.0$ ;  $\diamond$ : case 4 —  $\tau = 60.0$ , T = 1.0, a = 0.2,  $w_1 = 0.1$ ,  $w_2 = 0.3$ ,  $w_3 = 0.0$ ;  $\diamond$ : case 4 —  $\tau = 60.0$ , T = 1.0, a = 0.2,  $w_1 = 0.1$ ,  $w_2 = 0.3$ ,  $w_3 = 0.0$ 



Figure 5. Average packet delay (expressed in multiples of the packet transmission time  $\tau$ ) as a function of the number of nodes for the CSMA/CF protocol. +: case 1 -  $\tau$  = 15.0, T = 1.0, a = 0.1, w<sub>1</sub> = 0.1, w<sub>2</sub> = 0.3, w<sub>3</sub> = 0.0;  $\diamond$ : case 2 -  $\tau$  = 30.0, T = 1.0, a = 0.1, w<sub>1</sub> = 0.1, w<sub>2</sub> = 0.2, w<sub>3</sub> = 0.0;  $\star$ : case 3 -  $\tau$  = 60.0, T = 1.0, a = 0.1, w<sub>1</sub> = 0.1, w<sub>2</sub> = 0.3, w<sub>3</sub> = 0.0;  $\diamond$ : case 4 -  $\tau$  = 60.0, T = 1.0, a = 0.2, w<sub>1</sub> = 0.1, w<sub>2</sub> = 0.3, w<sub>3</sub> = 0.0



Figure 6. Average utilization factors of comparable CSMA/CD and CSMA/CF networks (for all cases  $\tau = 30.0$ , T = 1.0, a = 0.1);  $\blacksquare$ : CSMA/CF ( $w_1 = 0.1$ ,  $w_2 = 0.3$ ,  $w_3 = 0.0$ );  $\diamond$ : CSMA/CD ( $\delta = 1.0$ ); +: CSMA/CD ( $\delta = 0.8$ );  $\diamond$ : CSMA/CD ( $\delta = 0.5$ )

The authors intend to use this new protocol in the H-Network<sup>5, 6</sup>. The H-Network is a fast (14 Mbyte/s) LAN, used as a global communications pathway in the homogeneous multiprocessor<sup>15</sup>. There it serves packet traffic between distant processors co-operating in the computation of a parallel application. The parameters used in the computation of the average utilization factor and the average packet delay were chosen so as to conform with the projected implementation of the protocol on the H-Network.

#### ACKNOWLEDGEMENTS

This work was supported by grants from the Natural Sciences and Engineering Research Council, Canada; the Fonds pour la formation de Chercheurs et l'aide a la Recherche; and the Centre de Recherche Informatique de Montreal.

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### APPENDIX: CSMA/CF PROTOCOL CORRECTNESS

Denote by  $w_1^i$ ,  $w_2^i$ ,  $w_3^i$ ; i = 0, 1, 2, ..., N - 1 the three delay times, as explained above, for the i<sup>th</sup> ready station. Also, denote by  $t_i$  the arrival time (i.e. the time at which the i<sup>th</sup> station  $s_i$  first checks the contents of the vessel). Observe that  $t_0 < t_1 < ... < t_{N-1}$ . Assume also that the first arriving station  $s_0$ , at time  $t_0$ , finds the vessel (i.e. the contention channel) empty. Then, according to the CSMA/CF protocol, it deposits a pebble at time  $t_1^0 =$  $t_0 + w_1^0$ . All other stations will notice this event at time  $t_1^0 + a$  the latest, where a is the maximum channel propagation delay. With respect to the first arriving station the blocking time is denoted as  $t_{BLOCK}^0 = t_0 + w_1^0 + a$ . Any station with arrival time  $t_i > t_{BLOCK}^{o}$ , will see that there is at least one pebble in the vessel, and it will be blocked. On the other hand, stations with arrival times within  $[t_0,$  $t_{BLOCK}^{o}]$  will sense the vessel as being empty, and will deposit their pebbles at times  $t_1^i = t_i + w_1^i$ . When the first arriving station  $s_0$  re-checks the vessel at time  $t_2^0 =$  $t_0 + w_1^0 + w_2^0$ , it must ensure that all the other stations which sensed the vessel as being empty at times  $t_i$  $\in [t_0, t_{BLOCK}^0]$  had enough time to deposit their own pebbles, plus a propagation delay a. Thus,  $t_2^0 > t_1^i + a$ , or:

$$t_0 + w_1^0 + w_2^0 > t_i + w_1^i + a \tag{A1}$$

and for the limiting case, where  $t_i = t_{BLOCK}^0 = t_0 + w_1^0 + a$ relation (A1) gives  $w_2^0 > w_1^i + 2a$ . Denoting as  $w_1 = \max$ 

 $\{w'_1\}$  and generalizing for all the delays  $w'_2$ , we have:

$$w_2^i > w_1 + 2a \tag{A2}$$

Assuming that relation (A2) is satisfied it can be proven that the protocol is correct, i.e. no two stations are allowed to transmit at the same time. Indeed, assume that station  $s_i$  and  $s_j$  are transmitting at the same time. Assume also, without loss of generality, that station  $s_i$  arrives before station  $s_j$ , i.e.  $t_j < t_j$ . In order for station  $s_j$  to be able to contend for the contention channel, it should not be blocked by a station arriving before it, and certainly not by station  $s_j$ . Thus:

$$t_j < t'_{BLOCK} = t_j + w'_1 + a. \tag{A3}$$

Also, in order for both stations  $s_i$  and  $s_j$  to become successful at the same time, neither of them should sense the presence of the other when they check the vessel for the second time. Thus,  $s_i$  should not detect the presence of  $s_j$  when it checks the vessel at time  $t_2^i = t_i + w_1^i + w_2^i$ while  $s_j$  should not detect the presence of  $s_i$  at time  $t_2^i = t_i + w_1^j + w_2^i$ .

Stations  $s_i$  and  $s_j$  deposit their pebbles at times  $t_1^i = t_i + w_1^i$  and  $t_2^i = t_j + w_1^j$  respectively. Therefore, accounting for the propagation delays we have:

$$t_2^i < t_1^i + a \tag{A4}$$

$$t_2^i < t_1^i + a \tag{A5}$$

or 
$$t_i + w_1^i + w_2^i < t_i + w_1^i + a$$
 (A6)

and 
$$t_i + w_1^j + w_2^j < t_i + w_1^j + a$$
 (A7)

Combining relations (6) and (7),  $w_2^i + w_2^i < 2a$  results, which contradicts relation (A2).